

A Dynamic Interference Model for Benham's Top

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Abstract

It remains a mystery why colors emerge from the black-and-white pattern of a Benham's top. This article is an extension of one I published in 1979. Here I discuss in greater depth the reasons for manifestations of subjective color, with a focus on my own hypothesis, namely, that two successive stimuli are shifted in phase as they pass through the human visual system, causing dynamic interference that results in the emergence of a specific color. I hope this hypothesis will significantly contribute to the nearly 200-year history of attempts to elucidate subjective color.

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1. Introduction

The study of subjective color in perceptual psychology began with Prévost in 1826, followed by Fechner in 1839 and Benham in 1894¹⁾. Charles Benham (1860–1929) invented Benham's top, in which colors emerge from a black-and-white design. This toy became very popular in England and was featured in the scientific journal *Nature*, but even a century later, the cause of the subjective color remains unknown²⁾.

I studied Benham's top in 1979 and proposed a hypothesis regarding the appearance of subjective color³⁾. That article has been summarized and published in several English-language papers⁴⁻⁶⁾. I have since noticed, however, that my hypothesis was inadequately explained and difficult to understand. It has been over 40 years since my first investigation, but a reexamination has convinced me that my hypothesis is correct. Namely, I suggest that Benham's top causes two successive stimuli to pass through the human visual system, resulting in dynamic interference. In this article, I therefore attempt to explain as carefully and concisely as possible the process by which subjective color appears through the superposition of two out-of-phase stimuli.

2. Static Interference

I speculated that Benham's top somehow causes something like what in physics is called light interference.

Light has the characteristics of both particles and waves. Thomas Young (1773–1829) demonstrated the wave nature of light by showing that light passed through two slits produces interference fringes. When the crests or troughs of two waves overlap, the waves reinforce each other, while overlapping crests and troughs cancel each other out. The brightness

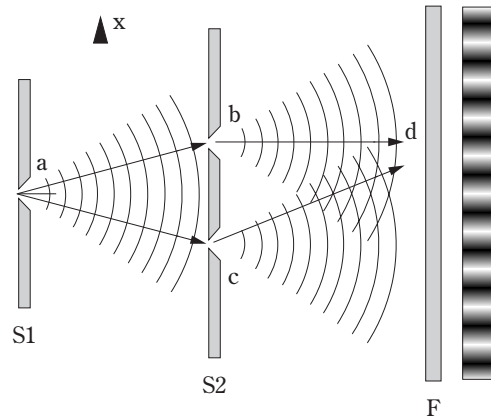


Fig. 1: Young's interference experiment.

(Source: https://en.wikipedia.org/wiki/Young%27s_interference_experiment#/media/File:Ebohr1_IP.svg © User:Stannered / Wikimedia Commons / CC-BY-SA 3.0 license)

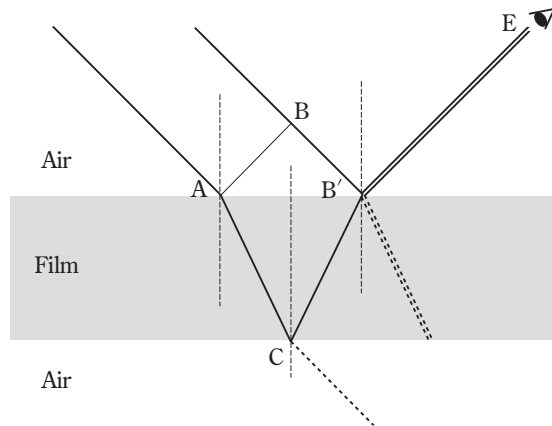


Fig. 2: Interference from a film (soap bubble).

increases where the two intensify each other, and decreases where they cancel, creating striped light and dark patterns on a screen.

As shown in **Fig. 1**, light emanating from slit *a* of sheet *S1* splits at slits *b* and *c* of sheet *S2*, then arrives at *d* on screen *F*. When the difference in the paths from *b* and *c* to *d* is an integer multiple of the light wavelength λ , there is constructive interference that strengthens the light intensity, and when the difference is half the wavelength, there is destructive interference that weakens it, producing a striped pattern of light and dark areas on screen *F*.

The thin film of a soap bubble produces rainbow colors due to interference between light reflected from the film's top surface and light reflected from its bottom. For example, consider **Fig. 2**, in which two parallel light beams *A* and *B* are obliquely incident to a thin film. One beam of light refracts at the film's top surface at *A*, reflects off the bottom surface at *C*, then refracts again at *B'* of the top surface to reach the eye at *E*, giving the path $A \rightarrow$

$C \rightarrow B' \rightarrow E$. Another beam of light simply reflects off the top surface at B' , tracing the path $B \rightarrow B' \rightarrow E$. The path difference is thus $\overline{AC} + \overline{CB'} - \overline{BB'}$. When that difference is an integer multiple of the light wavelength λ , the waves intensify, and the color of that wavelength is perceived. The result is rainbow colors.

Young's interference experiment and interference from thin films both involve *path differences* of light. Benham's top, however, produces nothing corresponding to path differences. Also, while Young's interference fringes and the iridescence of soap bubbles can be photographed, the colors produced by Benham's top cannot be captured on film because they are subjective colors; taking a photograph produces only a blurred black-and-white image.

Colors from Benham's top can be subjectively perceived by only humans through the human visual system, from the eyeball through the retina and then to the brain. Interference must be occurring somewhere, but we have been unable to locate exactly where. To distinguish between the interference seen in Young's experiment and that produced by Benham's top, we will refer to the former as "static interference" and the latter as "dynamic interference." We will discuss dynamic interference in detail.

3. From Polar to Cartesian Coordinates

I will now give an overview of Benham's Top. A typical example is a disk with a diameter of around 10 cm, one half painted solid black and the other having arcs of three lines each (**Fig. 3**). There are four blocks of arcs, each with a 45° central angle. Blocks can be created with two lines each, but using three makes the effect easier to see.

Giving the top a fast spin, at first you see nothing, but as its rotational speed slows the pattern begins to flicker, and colors immediately appear. When spinning the top clockwise (to the right), colors appear in the order blue, green, orange, and red from the periphery inward (**Fig. 4-1**).



Fig. 3: Benham's top.

The colors are not vivid, but they are clearly perceived as colors, not black or white. Spinning the top in the counterclockwise direction results in the color order red, orange, green,

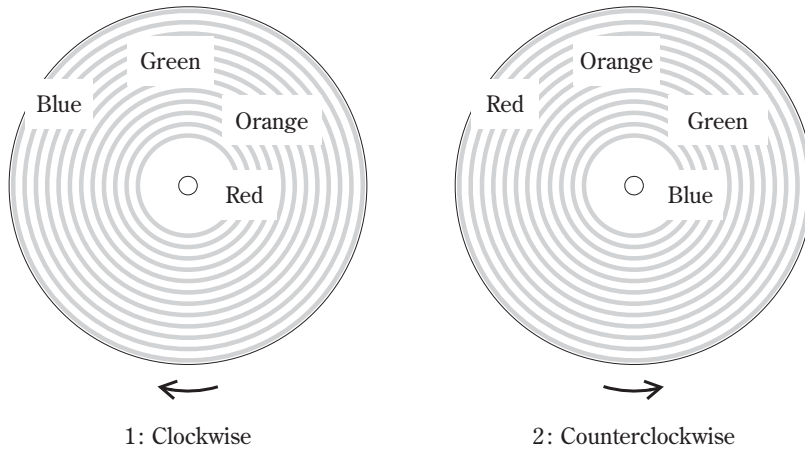


Fig. 4: Rotational directions and orders of colors.

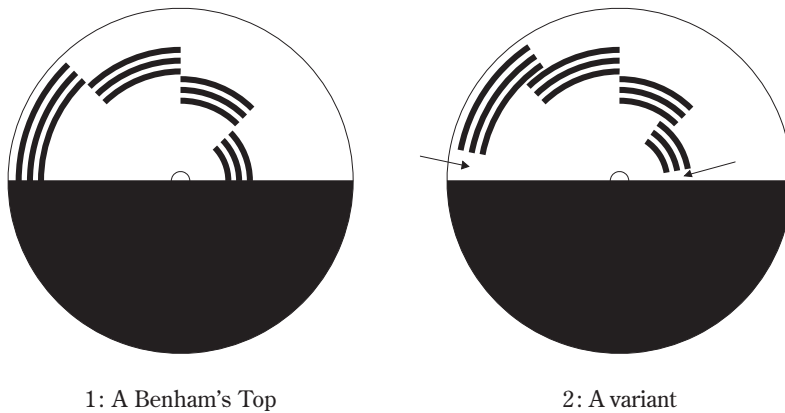


Fig. 5: A variant of Benham's top.

blue (**Fig. 4-2**). The fact that reversing the direction of rotation reverses the color order, and that the colors are arranged in the order of light wavelengths, like in a rainbow, are keys to elucidating subjective color.

The effect where a fan appears to be rotating in the opposite direction is due to a strobing effect, which can only be seen under fluorescent lighting, but colors appear on Benham's top even in sunlight. From this, we can infer that subjective color is closely related to the human visual system.

I focused on the fact that green and orange appear in the middle of the four arc blocks, not on the periphery or in the center. Why do those colors flip when the top spins in opposite directions? In the Benham's top shown in **Fig. 5-1**, setting aside the black lower half, the arrangements of the green and orange arcs are radially reversed. The central angle is 360° in one rotation and the black in the lower half covers 180° . When the 180° in the upper half is divided into white, black, and white, we have clockwise



Fig. 6: Conversion from polar to Cartesian coordinates.

Green arc: Black (180°) — White (45°) — Black (45°) — White (90°)

Orange arc: Black (180°) — White (90°) — Black (45°) — White (45°)

The pattern is black–white–black–white, and we can assume that the spacings of black and white are related to the color difference. For future development of this theory, I shifted the positions of the outer and central arcs in the standard Benham's top by about 20° around the central angle, as shown in **Fig. 5-2**, so that they do not touch the black area of the lower semicircle at the locations indicated by the arrows. In this way, all four blocks will have a black–white–black–white pattern, and the subjective colors, which previously had a grayish cast, become easier to see. This variant does not cause any unwanted effects.

We will continue our discussion using the pattern in **Fig. 5-2**. Converting polar to Cartesian coordinates, we get the pattern shown in **Fig. 6**. Following the top's periphery counter-clockwise from A to B, we can label the arcs as 1 through 4. We assume the disk is turning to the right (clockwise).

Fixing the viewpoint at the leftmost position A, the view first passes through the black area from A to B. This black region allows the eyes to rest and is unrelated to subjective color. After the disk completes a half revolution, arc 1 appears, followed by arcs 2 through 4. Each arc has three lines, but that is only to make the subjective colors easier to see and has no fundamental effect. From the center of the disk, the subjective colors are red for arc 1, orange for arc 2, green for arc 3, and blue for arc 4. In Cartesian coordinates (the right side of **Fig. 6**), the colors are red, orange, green, and blue from top to bottom.

Each of arcs 1 through 4 has the order black–white–black–white. In the Cartesian coordinates, the lengths of the first and second black parts are the same, but the lengths of the white parts before and after the second black part are different. If the first white provides a primary stimulus and the second a secondary stimulus, the two may produce some form of interference (reinforcement, cancellation, etc.).

For paints, black and white are equally regarded as colors, but strictly speaking, black is not a color. Black is an absorption of all colors, while white is their reflection. Thus, we must keep in mind that it is the white areas, not the black ones, that should be relevant to subjective color.

4. Dynamic Interference

Benham's top was found to exhibit subjective color when black–white–black–white patterns are repeated, like in the Cartesian coordinates in the right side of **Fig. 6**¹⁾. It was also found that the white areas, not the black areas, are related to subjective color. We can thus imagine that the first and second whites are likely to cause interference.

The interference seen in Young's interference experiment (**Fig. 1**) and in the thin film of a soap bubble (**Fig. 2**) was due to differences in the paths via which light arrives. In the case of Benham's top, however, visual stimuli travel to the visual system from the eyes, then through the retina and on to the brain, where they are perceived as subjective color, with nothing along that route corresponding to a path difference. Everything follows the same route, so interference seemingly cannot occur.

I will therefore show that here, interference occurs as a "phase difference" rather than "path difference." I refer to this as "dynamic interference," because it is interference that occurs by passing through the human visual system.

In the black–white–black–white pattern of a Benham's top, the white segments are where white light reflects and enters the eye (stimulates the visual system). We will thus refer to a first white segment as a primary stimulus, and to a subsequent white segment, following a black one, as a secondary stimulus. Benham's top rotates 2–5 times per second, and since this pattern is repeated, we can regard it as an irregular rectangular digital wave.

White light from a primary stimulus is absorbed by cones in the retina and perceived as white in the cerebrum. The same occurs as a result of the white light in a secondary stimulus. When these perceptions of white and white overlap, only white appears and there seems to be no room for other colors to emerge.

The basic unit of information processing by the nervous system of the human body is the neuron. Information transmission between neurons occurs by ionic conduction, which can fail to create a pulse because a certain threshold is not met, or it can be delayed due to transmission lag. I suspect that subjective color is related to this property.

In 1979, I thought that colors might emerge when a primary stimulus was delayed for some reason, causing it to overlap with a secondary stimulus (**Fig. 7**)³⁾. I speculated that the magnitude of the primary stimulus delay might be related to the wavelength of the emerging color, and inversely proportional to the strength of the primary stimulus, but I did not arrive at a theory.

Now, let us focus on a certain color wave included in white light and show that a phase shift can cause two waves to reinforce or cancel each other.

Consider two sinusoidal waves having the same wavelength and amplitude and at phase difference α :

$$y_1 = \sin(\theta + \alpha)$$

$$y_2 = \sin \theta$$

Fig. 8 shows an overview of the overlap. The sine waves for the primary and secondary stimuli in the top row of that figure have the same amplitude and wavelength, and there are five cycles. When there is no phase difference ($\alpha = 0$), superposing the two waves produces

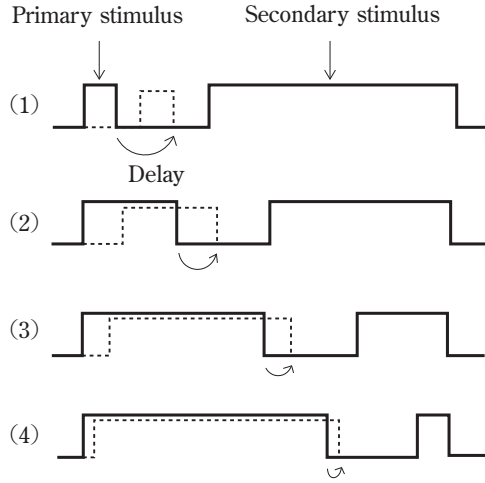


Fig. 7: Interference between primary and secondary stimuli.

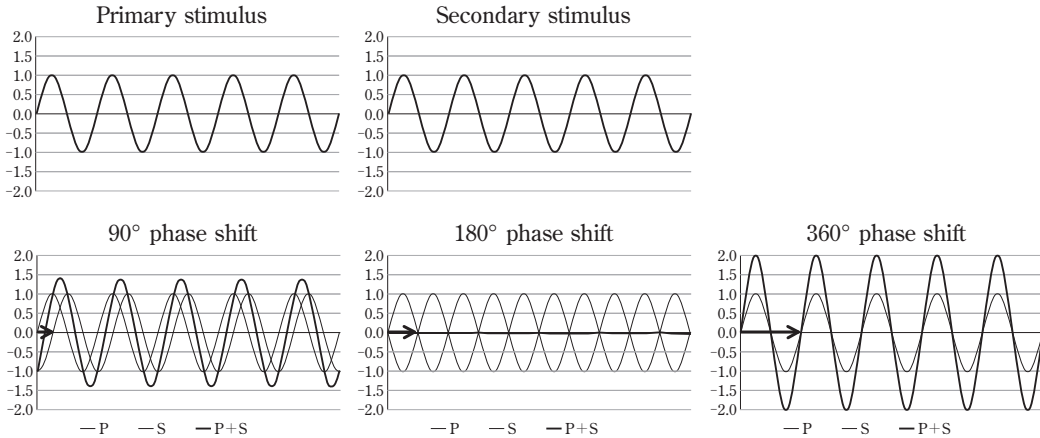


Fig. 8: Wave superpositions. (P: primary stimulus; S: secondary stimulus).

a sine wave with the same wavelength but twice the amplitude. When superposing the primary wave onto the secondary wave at a phase shift of 90° ($\alpha = \frac{\pi}{2}$), the maximum amplitude of the wave becomes 1.4 times larger, when shifted 180° ($\alpha = \pi$), the waves cancel and the amplitude becomes zero, and a 360° ($\alpha = 2\pi$) shift again doubles the amplitude.

By the formula for the sum-product of trigonometric functions, the superposition of two sine waves is

$$y_1 + y_2 = \sin(\theta + \alpha) + \sin \theta = 2 \cos \frac{\alpha}{2} \sin\left(\theta + \frac{\alpha}{2}\right).$$

Let us presume that when there is no phase difference between the waves providing the primary and secondary stimuli ($\alpha = 0$), no interference occurs: white on white remains white,

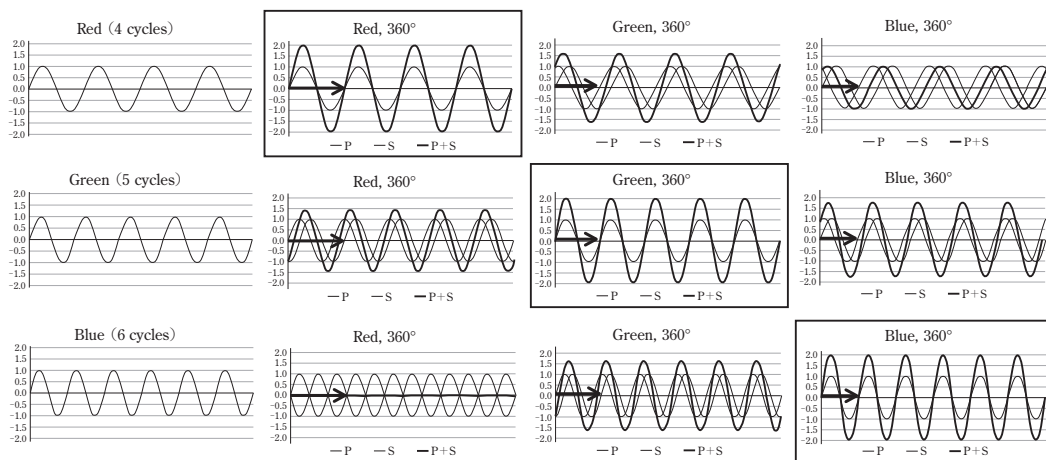


Fig. 9: Why certain colors emerge (P: primary stimulus; S: secondary stimulus).

and no color emerges. When there is a phase difference ($\alpha \neq 0$), interference occurs, producing a specific color.

White contains all colors, but to simplify the discussion, we consider only red, green, and blue. The human retina contains photoreceptor cells (cones) that sense color.

Fig. 9 is somewhat complex, but it shows twelve graphs in three rows and four columns. The leftmost column shows waves with four red cycles, five green cycles, and six blue cycles. Superposing all three would produce white. Each row shows superpositions of the waves providing primary and secondary stimuli with the phase shifted 360° at the red wavelength, 360° at the green wavelength, and 360° at the blue wavelength.

I will describe this in detail, using the four graphs in the first row. First, red (4 cycles) has twice the amplitude when shifted 360° at the red wavelength, 1.6 times the amplitude when shifted 360° at the green wavelength, and an unchanged amplitude when shifted 360° at the blue wavelength. Red thus only doubles in amplitude when shifted 360° at the red wavelength, and red emerges predominantly. That graph is highlighted with a frame.

Similarly, for the four graphs in the second row, green (5 cycles) has 1.4 times the amplitude when shifted 360° at the red wavelength, double the amplitude when shifted 360° at the green wavelength, and 1.7 times the amplitude when shifted 360° at the blue wavelength. Green thus only doubles in amplitude when shifted 360° at the green wavelength, where green predominantly emerges. That graph, too, is highlighted with a frame.

Finally, for the four graphs in the third row, blue (6 cycles) has zero amplitude when shifted 360° at the red wavelength, 1.6 times the amplitude when shifted 360° at the green wavelength, and double the amplitude when shifted 360° at the blue wavelength. Thus, blue doubles in amplitude only when shifted 360° at the blue wavelength, where blue predominantly emerges. Again, that graph is highlighted with a frame.

Now let's look at **Fig. 9** in the columnar (vertical) direction. The first column from the left superimposes red, green, and blue, producing white. White contains all colors.

In the second column, shifting and superimposing the red wavelength by 360° doubles the

red amplitude, increases the green amplitude by a factor of 1.4, and makes the blue amplitude become 0, making the red color predominantly emerge.

In the third column, shifting and superimposing the green wavelength by 360° increases the red amplitude by 1.6, doubles the green amplitude, and increases the blue amplitude by 1.6, making the green color predominantly emerge.

In the fourth column, shifting and superimposing the blue wavelength by 360° leaves the red amplitude the same, increases the green amplitude by 1.7, and doubles the blue amplitude, making the blue color predominantly emerge.

5. Equations of motion for delay

Fig. 9 shows why certain colors emerge, but let us consider how this corresponds to a Benham's top. In **Fig. 6** the rotating disk is converted to Cartesian coordinates. This causes the three arcs to become three straight lines, but I used three lines only to make the subjective colors appear more readily. The same effect would occur with a single line, which can be simplified as shown in **Fig. 10**.

As the top rotates, in Cartesian coordinates it moves from left to right, then repeats. From the top, 1 corresponds to red, 2 to orange, 3 to green, and 4 to blue. Those wavelengths are ordered from longest to shortest. In all cases, the order is black–white–black–white, where the first black segment is for resting the eye and unrelated to subjective color, and the subsequent white–black–white segments are related to subjective color.

I call the first white segment the primary stimulus and the second white segment, between the black ones, the secondary stimulus. Dynamic interference occurs when the primary stimulus shifts and overlaps the secondary stimulus. If the magnitude of the primary stimulus's shift is largest for Red (1) and smallest for Blue (4), that corresponds to the reason why a particular color emerges in **Fig. 9**. But does such a convenient hypothesis really hold up?

It is possible that the black segment between the two white segments functions to delay or inhibit the transmission of the primary stimulus. If the white segment providing the primary stimulus is significantly delayed when it is short, and only slightly delayed when it is long, that would conveniently explain subjective color. Therefore, I made a bold hypothesis.

Consider white primary stimuli as the amount of light, the substance being transmitted, and the object in motion, with masses $m(m_1, m_2, m_3, m_4)$ and accelerations $a(a_1, a_2, a_3, a_4)$.

If we consider the area of white as a physical quantity transmitted by white light to the visual system, we can compare its size to mass. If the white area is large, the amount of light is large and the mass is large; if that area is small, the amount of light and mass are small.

Black areas reflect no white light and convey nothing to the visual system. Rather, they may function to stop or delay the progression of physical quantities conveyed to the visual system by preceding white areas.

Black is thus a force that acts to delay white transmissions. Since the length of each black segment is the same, we can let the force be F (a constant), and we obtain for each the equation of motion:

$$F = ma = m_i a_i.$$

From **Fig. 10**, the length of the primary stimulus is the shortest for red (1) and longest for blue (4), so the mass m has the relation

$$m_1 < m_2 < m_3 < m_4.$$

Since the F in the equation of motion $F=ma$ is constant, the acceleration a has the relation

$$a_1 > a_2 > a_3 > a_4.$$

Newton’s equations of motion state that the lighter an object, the easier it is to move, and the heavier it is, the harder it is to move, and this also holds true for Benham’s top. Namely, the smaller the primary stimulus segment, the greater its displacement, and the larger it is, the smaller the displacement, causing overlapping with the secondary stimulus and producing dynamic interference.

This can be described for red (1) as follows: The mass m_1 of the primary stimulus is the smallest, so the acceleration a_1 is the largest, and thus the primary stimulus experiences the largest shift and overlaps and interferes with the secondary stimulus. As the first row in **Fig. 9** shows, the longest wavelength, red, emerges.

Similarly, the description for blue (4) is as follows: The mass m_4 of the primary stimulus is the largest, so the acceleration a_4 is the smallest, and thus the primary stimulus experiences the smallest shift and overlaps and interferes with the secondary stimulus. As shown by the third row in **Fig. 9**, the shortest wavelength, blue, emerges.

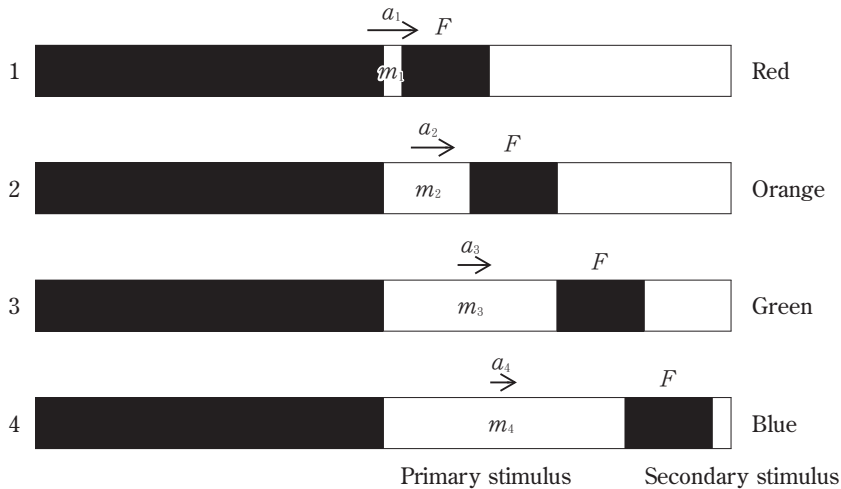


Fig. 10: Equation of motion for delay ($F=ma$).

6. Conclusion

From the above, we can deduce why colors emerge on Benham’s top.

It has been nearly two centuries since Prévost discovered subjective color in 1826. A number of black-and-white patterns have been devised from which subjective color appears, but the cause of this phenomenon remains unknown. In the human visual system, light produces two successive stimuli, a primary stimulus and a secondary stimulus, which are transmitted from the eye via the retina to the cerebrum. A phase shift between stimuli will cause dynamic

interference, resulting in the appearance of subjective colors.

I think this is ultimately a consequence of the model of subjective color expression shown in **Fig. 11**. Letting a , b , c , d be the respective lengths of the black–white–black–white segments, when this pattern is repeated n times per second, it seems that varying the values of these five variables produces a variety of colors. To test this, I would like to have not a spinning disk like a Benham's top, but an experimental apparatus that can display black–white–black–white patterns on a screen. Proving my hypothesis would be a major advancement in the study of subjective color, following Prévost, Fechner, and Benham^{1, 2)}.

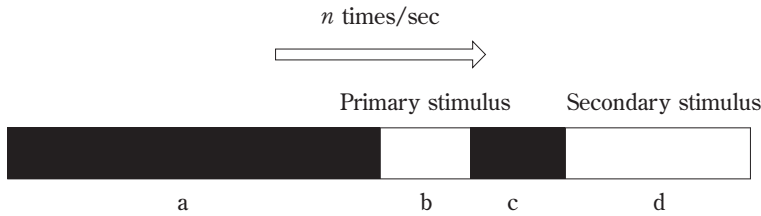


Fig. 11: A model of subjective color expression.

Appendix

I first became interested in Benham's top in 1979, when I wrote an article³⁾ for the IBM Japan public relations magazine *Mugendai* describing a paper by Cohen and Gordon¹⁾ along with my own discussion. Since the present discussion is an extension of that article, I hope you will carefully read that article in Japanese³⁾ or English⁵⁾ to understand an overview of Benham's top and subjective color.

I learned that Benham's top remained as an unsolved problem in the university library at Cambridge while I was studying there in 2005²⁾. After retiring from my university post, I was able to revisit this topic in 2023 and theorize my hypothesis.

Fig. 9 explains why certain colors emerge, and that result can be used to estimate subjective colors. Taking the three primary colors as RGB, the magnitude of the amplitude due to superposition in the columnar (vertical) direction in **Fig. 9** is as follows:

$$(R, G, B) = (2, 1.4, 0),$$

$$(R, G, B) = (1.6, 2, 1.6),$$

$$(R, G, B) = (1, 1.7, 2).$$

Recalculating these as chromaticity values of 0 to 255, we get

$$(R, G, B) = (255, 179, 0),$$

$$(R, G, B) = (204, 255, 204),$$

$$(R, G, B) = (128, 217, 255).$$

Fig. 12 shows a plot based on these figures.

The top row is $(R, G, B) = (255, 255, 255)$, which is white due to additive mixing of the three primary colors. The second row shows the subjective color with only the red wavelength phase-shifted, the third row shows the subjective color with only the green wavelength phase-shifted, and the fourth row shows the subjective color with only the blue wavelength phase-shifted. Subjective colors are intermediate colors, more like pastels than vivid reds, greens, and blues.

In **Fig. 4**, subjective colors are described as blue, green, orange, red, etc., but in reality, while the colors remain the same, they are indistinct.

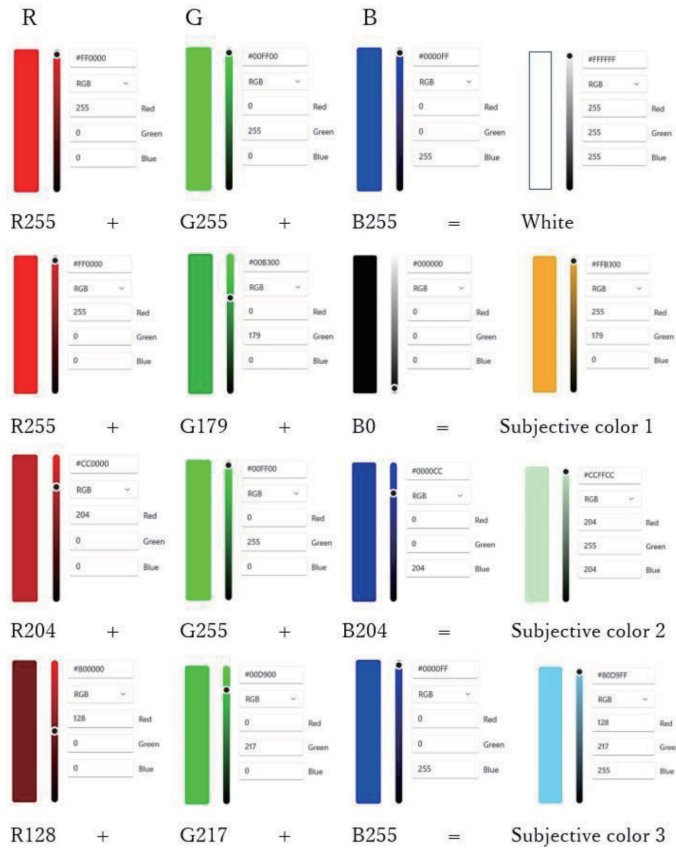


Fig. 12: Subjective color estimation with a dynamic interference model.

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