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Abstract

This study develops a unified framework for markets with network externalities that generalizes the traditional dichotomy between passive and responsive expectations. We demonstrate that, contrary to conventional wisdom, social welfare may increase when consumers form "inaccurate" (out-of-equilibrium) expectations, provided that expected demand is sufficiently price-elastic. These elastic expectations act as an endogenous constraint on monopoly pricing, mitigating the underproduction distortion through the logic of the theory of second best. We further show that although monopolist's profit is maximized under responsive expectations, social welfare is higher when expectations are highly elastic, revealing a conflict of interest between the firm and society. Extending the analysis to a differentiated duopoly, we show that the disciplinary mechanism is reinforced by a novel cross-expectation effect. The conflict of interest also acquires a richer structure: under competition, firms may strictly prefer passive expectations to responsive expectations equilibrium because inelastic consumer expectations soften price competition and alleviate the prisoner's dilemma in pricing.

Keywords: Monopoly, Network Externalities, Passive Expectations, Responsive Expectations, Self-fulfilling Expectations, Social Welfare
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1 Introduction

Network externalities are a defining feature of modern digital industries, from telecommunication standards and video game consoles to social networking platforms, in which product value rises as the number of adopters (i.e., network size) increases (Katz and Shapiro, 1985; Farrell and Saloner, 1985; Clements and Ohashi, 2005). In such markets, not only the actual network size but also consumers' *expectations* regarding future adoption are crucial determinants of pricing, technology diffusion, and welfare.[1] Consumers, in practice, often consider the observed launch price a signal of a network's future viability. A well-known illustration comes from the video game industry during the 1990s: the high launch price of the 3DO Interactive Multiplayer (approximately USD 700) signaled weak adoption prospects, which triggered overly pessimistic expectations among consumers and complementary software developers alike, ultimately leading to the product's commercial failure. This suggests that consumers' expectations of future demand are inherently price-dependent. Despite this, the theoretical literature has largely focused on two canonical but restrictive expectation regimes. Under *passive expectations*, consumers treat expected network size as fully exogenous when forming inferences from a firm's product pricing. Alternatively, under *responsive expectations*, consumers perfectly anticipate actual network adoption at every price level. Hurkens and López (2014) provide the landmark analysis demonstrating that the two regimes generate substantially different competitive outcomes. Subsequent contributions extend this binary comparison into various market structures (Hagiu and Halaburda, 2014; Toshimitsu, 2017; Nakamura, 2021). Toshimitsu (2017) compares passive and responsive expectations within a monopoly with network goods, finding that responsive expectations induce a lower price. Nakamura (2021) extends the comparison to a duopoly with both price and quantity competition. However, these contributions share a common structural restriction: expected network

[1] Building on the early work of Katz and Shapiro (1985) and (1986), as well as Farrell and Saloner (1985), the subsequent literature has explored network externalities across a wide range of settings, including the welfare economics of networks (Economides, 1996), international product standards (Barrett and Yang, 2001), nonlinear telecommunications pricing (Hahn, 2003), dynamic price competition (Cabral, 2011), and price competition within differentiated duopolies (Griva and Vettas, 2011). Haltiwanger and Waldman (1985) provide early evidence that even a small fraction of agents deviating from rational expectations can have a disproportionately large effect on equilibrium outcomes.

size is modeled as either a fixed constant (passive) or a perfect tracker of actual demand (responsive) without allowing for intermediate or continuous parameterization of expectation sensitivity. In particular, neither study addresses whether *intermediate* expectations lying between the two poles, passive and responsive, might dominate from a welfare perspective or characterizes the welfare-optimal degree of expectation accuracy. no existing study formalizes expected demand as a smooth price-dependent function arising from consumers ' inference about network's future success.

This paper fills these gaps by developing a generalized model that incorporates price-elastic expectations. We explicitly model how consumers adjust their forecasts of network size in response to price changes, encompassing both passive and responsive expectations as special or limiting cases. We adopt a reduced-form representation of expectations consistent with a Bayesian price-signaling framework (Spence, 1973; Milgrom and Roberts, 1986), in which self-fulfilling expectations are required only in equilibrium. We use the monopoly setting as our initial baseline to cleanly isolate how this expectation formation mechanism interacts with the firm's standard incentive to underproduce before extending the framework to a differentiated duopoly.

This generalization yields three primary findings. First, we show that a higher price elasticity of expected demand unambiguously improves social welfare. If consumers' expectations are highly elastic—meaning they substantially reduce their network size predictions when faced with a price increase—society benefits. The mechanism follows the logic of the theory of second best (Lipsey and Lancaster, 1956): consumers' out-of-equilibrium pessimism about network adoption introduces a "second distortion" that effectively counteracts the monopolist's primary tendency to underproduce, thereby lowering prices and expanding output. Specifically, highly elastic expectations imply that even a small price increase leads to a significant drop in expected network size, forcing the monopolist to maintain lower prices to secure demand. Second, the model reveals a conflict of interest between the firm and society. Whereas the monopolist's profit is maximized under responsive expectations (under which consumers correctly anticipate the actual network size), social welfare is strictly higher when expectations are highly elastic. Third, extending the analysis to a differentiated duopoly reveals a novel *cross-expectation effect* that reinforces this disciplinary mechanism. Furthermore, the conflict

of interest becomes more complex under market competition: firms may strictly prefer that consumers hold passive (inelastic) rather than responsive expectations because inelastic expectations soften price competition and mitigate the prisoner's dilemma in pricing.

These findings have policy implications that run counter to conventional thinking. A common assumption in policy discussions is that improving the accuracy of consumer information about network adoption is socially beneficial. Indeed, the view that network externalities necessarily generate market failures has been questioned (Liebowitz and Margolis, 1994). Our analysis suggests that in environments characterized by substantial market power and network externalities, policies aimed at aligning consumer expectations with reality may reduce the endogenous constraint generated by out-of-equilibrium expectations on monopoly pricing, thereby worsening the underproduction problem and reducing social welfare.

The remainder of this paper is organized as follows. Section 2 introduces the model and the generalized expectation formation mechanism. Section 3 analyzes equilibrium prices, welfare, and profits under alternative expectation regimes, along with a linear demand illustration. Section 4 extends the framework to a differentiated duopoly. Section 5 concludes.

2 The Model of Network Externalities and Expectations

This section introduces the model and the generalized expectation formation mechanism. The key departure from the existing literature is that we allow expected network size to depend smoothly on price. As mentioned, under the passive expectations framework (Katz and Shapiro, 1985), expected network size is treated as exogenous to firms' pricing decisions. Under responsive expectations (Hurkens and López, 2014), expected network size coincides with actual demand at each price level. The approach adopted here generalizes both frameworks by allowing expected network size to respond to price changes while requiring expectations to be fulfilled only in equilibrium. To our knowledge, no existing static model of network externalities incorporated the price-dependent formulation

$x^e(p)$ proposed below.

We now describe consumer behavior to derive the demand function. Assume a continuum of consumers characterized by a taste parameter θ, which is distributed according to the density function $f(\theta)$ and the cumulative distribution function $F(\theta)$. By purchasing one unit of the good, a consumer obtains a net surplus:

$$U(\theta) = \theta + vX^e - p,$$

where the first term denotes intrinsic utility, the second term captures the network externality, and p denotes the price of the good. The parameter $v \in [0, 1]$ measures the degree of the network externality, and X^e denotes consumers' expected network size.

We propose a generalized expectation formulation, $X^e = \alpha x^e(p) + (1 - \alpha)x$, where $x^e(p)$ denotes the expected demand when the firm sets price p, and x denotes actual demand. Unlike the rational expectations benchmark, $x^e(p)$ need not coincide with actual demand for all price levels. However, in equilibrium, expectations must be fulfilled: $x^e(p^*) = x[p^*, x^e(p^*)]$.

The expectation function $x^e(p)$ provides a reduced-form description of how consumers form expectations about network size in response to firms' pricing decisions. This reduced form can be grounded within a Bayesian signaling framework: if the firm possesses private information about the product's fundamental viability and consumers use the observed price as a signal to update their posterior beliefs about network adoption, $x^e(p) = \mathbb{E}[\tilde{X} \mid p]$ emerges as a smooth, price-dependent function with $x^{e\prime}(p) \leq 0$ (see Appendix A for a formal derivation). Rather than modeling the microfoundations of belief formation, this paper adopts $x^e(p)$ as a reduced-form expectation function consistent with such Bayesian updating.

The parameter $\alpha \in \{0, 1\}$ determines the expectation regime. When $\alpha = 0$, consumers' expectations coincide with realized network size, so their expectations coincide with actual demand at every price. This corresponds to the responsive expectations case. When $\alpha = 1$, demand depends entirely on the price-contingent expectation function $x^e(p)$, which need not equal actual demand off the equilibrium path. In particular, when $\alpha = 1$ and $x^{e'} = 0$, price has no influence on consumer expectations, corresponding to

the passive expectations regime.

Remark 1 (Nested Structure of Expectation Regimes)**.** The generalized framework under $\alpha = 1$ encompasses the responsive expectations regime ($\alpha = 0$) as a special case. To see this, suppose consumers possess perfect foresight even off the equilibrium path, such that their expected demand exactly matches actual demand at every price level: $x^e(p) \equiv x[p, x^e(p)]$ for all p. Differentiating both sides with respect to p, and solving for dx/dp, yields exactly the same self-referential demand system that characterizes the $\alpha = 0$ regime. This nesting property justifies our focus on the $\alpha = 1$ case as the more general framework in the subsequent analysis.

Let $u = vX^e - p$ be the net surplus excluding the standalone benefit (henceforth, *the common net surplus*) and $\bar{u}$ be the reservation utility. To simplify the firm's profit maximization problem, we formulate it as choosing u rather than p. Given these assumptions, all consumers satisfying $\theta \geq \bar{u} - u$ purchase the good. Therefore, the number of consumers who purchase $n(u)$ is given by $n(u) = 1 - F(\bar{u} - u)$, so actual demand is $x = n(u)$. Note that $n(u)$ is strictly increasing: $n'(u) = f(\bar{u} - u) > 0$. Letting $p = D(u)$ denote the solution for $vX^e - p = u$ for p, we find that

$$\frac{dp}{du} = D'(u) = \frac{(1-\alpha)vn'(u) - 1}{1 - \alpha v x^{e'}} < 0. \tag{1}$$

Assuming that monopoly profit is $\pi = n(u)D(u) - C(n(u))$, where $C(\cdot)$ denotes the cost function with marginal cost $C'(x) > 0$, the monopoly problem is $\max_u \pi(u)$. The first-order condition is

$$\begin{aligned} & n'(u)\left[p - C'(n(u))\right] + n(u)D'(u) = 0 \\ \Leftrightarrow\ & p = C'(n(u)) - \frac{n(u)}{n'(u)}D'(u). \end{aligned} \tag{2}$$

Thus, the monopolist prices above its marginal cost since $n'(u) > 0$ and $D'(u) < 0$. Throughout the analysis, we assume the standard second-order condition $\pi''(u^*) < 0$, which ensures that the first-order condition (2) characterizes a unique interior maximum. The precise condition and its verification for standard demand specifications are provided in Appendix D.

The markup, measured by the Lerner index, satisfies the inverse elasticity rule:

$$\frac{p - C'(n(u))}{p} = -\frac{n(u)D'(u)}{n'(u)p} = \frac{1}{\epsilon},$$

where $\epsilon \equiv -(dn(u)/dp)(p/n(u))$ is the price elasticity of demand.

From (1) and (2), the equilibrium prices under $\alpha = 0$ (responsive expectations) and $\alpha = 1$ (including passive expectations) are

$$p^{(0)} = C'(n(u)) - vn(u) + \frac{n(u)}{n'(u)} \quad \textit{if} \;\; \alpha = 0,$$
$$p^{(1)} = C'(n(u)) - \frac{n(u)}{n'(u)(vx^{e'} - 1)} \quad \textit{if} \;\; \alpha = 1.$$

3 Equilibrium Analysis and Welfare Implications

This section analyzes the implications of the parameter α by comparing the equilibrium monopoly prices and social welfare derived in the previous section.

3.1 Monopoly Pricing under Different Expectation Regimes

Using the monopoly pricing rules $p^{(0)}, p^{(1)}$ and $dp/du = -1/(1 - vx^{e'})$ under $\alpha = 1$, the price difference can be expressed as

$$\begin{aligned} p^{(1)} - p^{(0)} &= vn(u)\left[1 - \frac{x^{e'}}{n'(u)(vx^{e'} - 1)}\right] \\ &= vn(u)\left[1 - \epsilon^e\left(-\frac{dp}{dn(u)}\frac{x^e}{p}\right)\right], \end{aligned}$$

where $\epsilon^e \equiv -(px^{e'})/x^e$ denotes the price elasticity of expected demand under $\alpha = 1$. Assuming that expectations are fulfilled in equilibrium (i.e., $x^e = n(u)$), the relationship simplifies to

$$p^{(1)} - p^{(0)} = vn(u)\left[1 - \frac{\epsilon^e}{\epsilon}\right].$$

Proposition 1. *Suppose consumers hold expectations about network size under either $\alpha = 0$ or $\alpha = 1$. If the price elasticity of expected demand ϵ^e is lower (higher) than the price elasticity of actual demand ϵ, the equilibrium price $p^{(1)}$ is higher (lower) than the price under responsive expectations ($\alpha = 0$). That is, $p^{(1)} > (<) p^{(0)}$.*

Proposition 1 demonstrates that the equilibrium price depends critically on the price elasticity of expected demand. When expected demand is highly elastic, the monopolist faces a strong consumer response to price changes and is thus forced to set a lower price. In contrast, when expected demand is inelastic, consumers expect network size to remain stable despite price increases, allowing the monopolist to charge a higher price. In particular, when consumers hold purely passive expectations ($\alpha = 1$ and $x^{e'} = 0$), we have $\epsilon^e = 0$, which always yields $p^{(1)} > p^{(0)}$. This is consistent with the findings of Hurkens and López (2014), who argue that competition becomes fiercer under responsive expectations because firms can no longer exploit consumer unresponsiveness as they can under passive expectations.

To understand the underlying mechanism, consider the first derivative of the firm's profit with respect to the common net surplus u:

$$\frac{d\pi}{du} = n'(u)\left[p - C'(n(u))\right] + n(u)\frac{dp}{du}.$$

The first term captures the positive volume effect of an increase in u, which is identical in both regimes. The second term represents the negative margin effect: increasing u requires the firm to reduce p. Under $\alpha = 0$, $dp/du = vn'(u) - 1$; because the network effect perfectly reflects actual demand, an increase in u automatically boosts network value, partially offsetting the required reduction in p. Under $\alpha = 1$, $dp/du = -1/(1 - vx^{e'})$; here, the incentive to increase u is determined entirely by $|x^{e'}|$. When $x^{e'} = 0$, $dp/du = -1 < vn'(u) - 1$, so $p^{(1)}$ is unambiguously higher than $p^{(0)}$.

The intuition behind these results and their effects is closely tied to how consumers interpret price changes as favorable or unfavorable signals of network participation. If expected demand is relatively inelastic, consumers believe the network size is robust to price hikes, allowing the monopolist to extract more surplus. If expected demand is highly elastic, consumers interpret a price increase as a strong signal that the network will shrink, thereby imposing stronger discipline on the monopolist's pricing power.

A key implication is that responsive expectations do not necessarily yield the most competitive market outcomes. The intensity of market discipline depends on the price elasticity of expected demand; if expected demand is sufficiently elastic, prices may fall

strictly below those under responsive expectations benchmark.

3.2 Social Welfare and the Second-Best Mechanism

Consumer surplus, $CS(u)$, is

$$CS(u) \equiv \int_{\bar{u}-u}^{\infty} (\theta + u - \bar{u}) f(\theta) d\theta,$$

with $CS'(u) = n(u) > 0$, so consumer surplus is strictly increasing in the common net surplus, u. Social welfare $W(u) = \pi(u) + CS(u)$ satisfies $dW/du = n(u) > 0$ at the monopolist's optimum, where $\pi'(u) = 0$. Thus, the monopolist always underprovides u relative to the social optimum. Any mechanism that induces the firm to increase u—equivalently, to lower the actual price p and expand output $n(u)$—unambiguously improves social welfare.

Proposition 2. *Suppose consumers hold expectations about the network size under $\alpha = 1$. A higher (lower) price elasticity of expected demand, ϵ^e, leads to higher (lower) social welfare.*

When the price elasticity of expected demand is low, Proposition 1 implies that $p^{(1)}$ is higher, which reduces the common net surplus u, consequently lowering actual output, $n(u)$, and social welfare. Conversely, when elasticity is high, the firm is constrained in its ability to raise prices aggressively, forcing a higher u and greater output.

Corollary 1. *Suppose consumers hold expectations under $\alpha = 1$. In the limiting case where expected demand becomes perfectly elastic with respect to price (i.e., $x^{e'} \to -\infty$), the equilibrium monopoly price converges to the marginal cost ($p^{(1)} \to C'(n(u))$), and the market outcome achieves the socially optimal (first-best) level.*

As $x^{e'} \to -\infty$, the marginal effect $D'(u) = -1/(1 - vx^{e'})$ converges to zero, so the monopolist's markup completely vanishes. This implies that when the price elasticity of expected demand becomes sufficiently high, social welfare under $\alpha = 1$ strictly exceeds that under responsive expectations ($\alpha = 0$).

Viewed through the lens of the theory of second best, our model demonstrates (Lipsey and Lancaster, 1956) that in a monopoly plagued by the primary distortion of under-

production, highly elastic consumer expectations introduce a countervailing "second distortion." Paradoxically, this inaccurate pessimism strongly disciplines the monopolist's pricing behavior, effectively replicating a perfectly competitive outcome within a pure monopoly market.

Note on the first-best benchmark. The first-best allocation in Corollary 1 refers to the social optimum in which a single price is charged to all consumers (the constrained first-best), defined by $p = C'(x)$. In the presence of network externalities, however, the unconstrained social optimum also requires internalizing the positive externality generated by each consumer's adoption, which would call for a subsidy vx^* per unit. Our result shows that elastic expectations drive the *monopoly price* to marginal cost. As a result, the competitive benchmark is replicated and the underproduction distortion disappears, even though the network externality itself is not internalized. Accordingly, we use the term "first-best" throughout to refer to the competitive (price-equals-marginal-cost) outcome rather than the Pareto optimum that fully internalized network externalities.[2]

3.3 Firm Profit and Conflict of Interest

While highly elastic expectations are desirable from a social welfare perspective, an examination of the monopolist's preferences reveals a conflict of interest.

Proposition 3. *Equilibrium monopoly profit under responsive expectations ($\alpha = 0$) is always greater than or equal to equilibrium profit under $\alpha = 1$, regardless of the price elasticity of expected demand ϵ^e. Furthermore, under $\alpha = 1$, equilibrium profit exhibits an inverted-U-shaped relationship with ϵ^e: it strictly increases for $\epsilon^e < \epsilon$, attains its maximum $\pi^{(0)}$ at $\epsilon^e = \epsilon$, and strictly decreases for $\epsilon^e > \epsilon$.*

Proof. See Appendix B. □

From the firm's perspective, the ideal environment is $\alpha = 0$. In this regime, consumers correctly perceive the actual network size, allowing the firm to fully internalize the network externality and extract maximum monopoly rent. Under $\alpha = 1$, the firm's

[2]Formally, the unconstrained social optimum satisfies $p = C'(x) - vx$, which incorporates the marginal social value of network expansion. Corollary 1 instead targets the competitive benchmark $p = C'(x)$, the standard welfare reference in the network externalities literature (see Katz and Shapiro, 1985).

profit is maximized precisely when $\epsilon^e = \epsilon$, replicating the $\alpha = 0$ outcome. For $\epsilon^e \neq \epsilon$ in either direction, profit falls below $\pi^{(0)}$: when $\epsilon^e < \epsilon$, the firm raises its price, but the resulting contraction in output more than offsets the higher margin; when $\epsilon^e > \epsilon$, the firm is disciplined into a lower price, reducing margins. Thus, the firm's profit is an inverted-U function of ϵ^e, bounded above by $\pi^{(0)}$.

Social welfare favors the exact opposite: welfare is maximized under $\alpha = 1$ with highly elastic expectations. The very mechanism that reduces the firm's profit—consumers' intense pessimism regarding network collapsing—mitigates the fundamental problem of underproduction. In summary, the firm prefers $\alpha = 0$; under $\alpha = 1$, profit exhibits an inverted-U in ϵ^e, while society's welfare is strictly increasing in ϵ^e and admits no interior maximum. This divergence implies that public policy aimed at improving welfare in network industries should encourage greater price sensitivity in consumer expectations, even if such expectations are technically inaccurate away from the equilibrium path.

3.4 An Illustration with Linear Demand

To provide a concrete illustration, we specify a linear demand model: $x = a - b(p - vX^e)$, with $a > 0$, $b > 0$, and constant marginal cost $c \geq 0$. To ensure a positive network effect that does not explode, we assume $v > 0$ and $1 - vb > 0$. Let $e \equiv -\partial X^e / \partial p \geq 0$ denote the absolute price sensitivity of expected demand. Applying the definitions of the absolute actual price sensitivities, $\Phi \equiv -dx/dp$ under each regime, we obtain:

$$\Phi^{(1)} = b(1 + ve), \quad \Phi^{(0)} = \frac{b}{1 - vb}.$$

In equilibrium, expectations are fulfilled ($X^e = x^*$). Substituting this condition into the demand function and the firm's first-order condition ($p^* = c + x^*/\Phi$) yields the following expression for the equilibrium price:

$$p^* = \frac{c\Phi(1 - vb) + a}{\Phi(1 - vb) + b}.$$

Assuming $a - cb > 0$ to guarantee positive output, it is straightforward to show that $\partial p^* / \partial \Phi < 0$. Furthermore, the monopolist's equilibrium profit is given by $\pi^* = (p^* - c)x^* = \Phi(p^* - c)^2$. This explicit solution directly confirms our general propositions:

- **Proposition 1:** $p^{(1)} < p^{(0)}$ iff $\Phi^{(1)} > \Phi^{(0)}$, i.e., $e > vb^2/(1-vb)$. Sufficiently elastic expectations therefore induce a strictly lower price.

- **Proposition 2:** An increase in e raises $\Phi^{(1)}$, lowers p^*, and expands output, unambiguously improving social welfare.

- **Proposition 3:** Differentiating π^* with respect to $\Phi^{(1)}$ yields $\partial\pi^*/\partial\Phi^{(1)} \propto b - \Phi^{(1)}(1-vb)$, which is positive for $\Phi^{(1)} < \Phi^{(0)}$, zero at $\Phi^{(1)} = \Phi^{(0)}$, and negative for $\Phi^{(1)} > \Phi^{(0)}$. Since $\Phi^{(1)} = b(1+ve)$ is strictly increasing in e, profit is an inverted-U shaped in ϵ^e, attaining its maximum $\pi^{(0)}$ at $\epsilon^e = \epsilon$, while social welfare is strictly increasing in ϵ^e, thereby confirming the conflict of interest.

- **Corollary 1:** As $e \to \infty$, $\Phi^{(1)} \to \infty$ and $p^* \to c$, thereby achieving the first-best allocation.

Figure 1 plots the equilibrium outcomes against $\epsilon^e \equiv e(p^*/x^*)$, which is endogenously determined as the parameter e varies. The top panel shows that for sufficiently large ϵ^e, the equilibrium price p^* ($\alpha = 1$) drops below the benchmark $p^{(0)}$ ($\alpha = 0$) and asymptotically approaches marginal cost c. The middle and bottom panels illustrate the divergence between social and private interests: social welfare W^* is strictly increasing (Proposition 2), while firm profit π^* is an inverted-U shape (Proposition 3), reaching $\pi^{(0)}$ at $\epsilon^e = \epsilon$ and declining thereafter. The fundamental conflict between private and social objectives arises only when $\epsilon^e > \epsilon$.

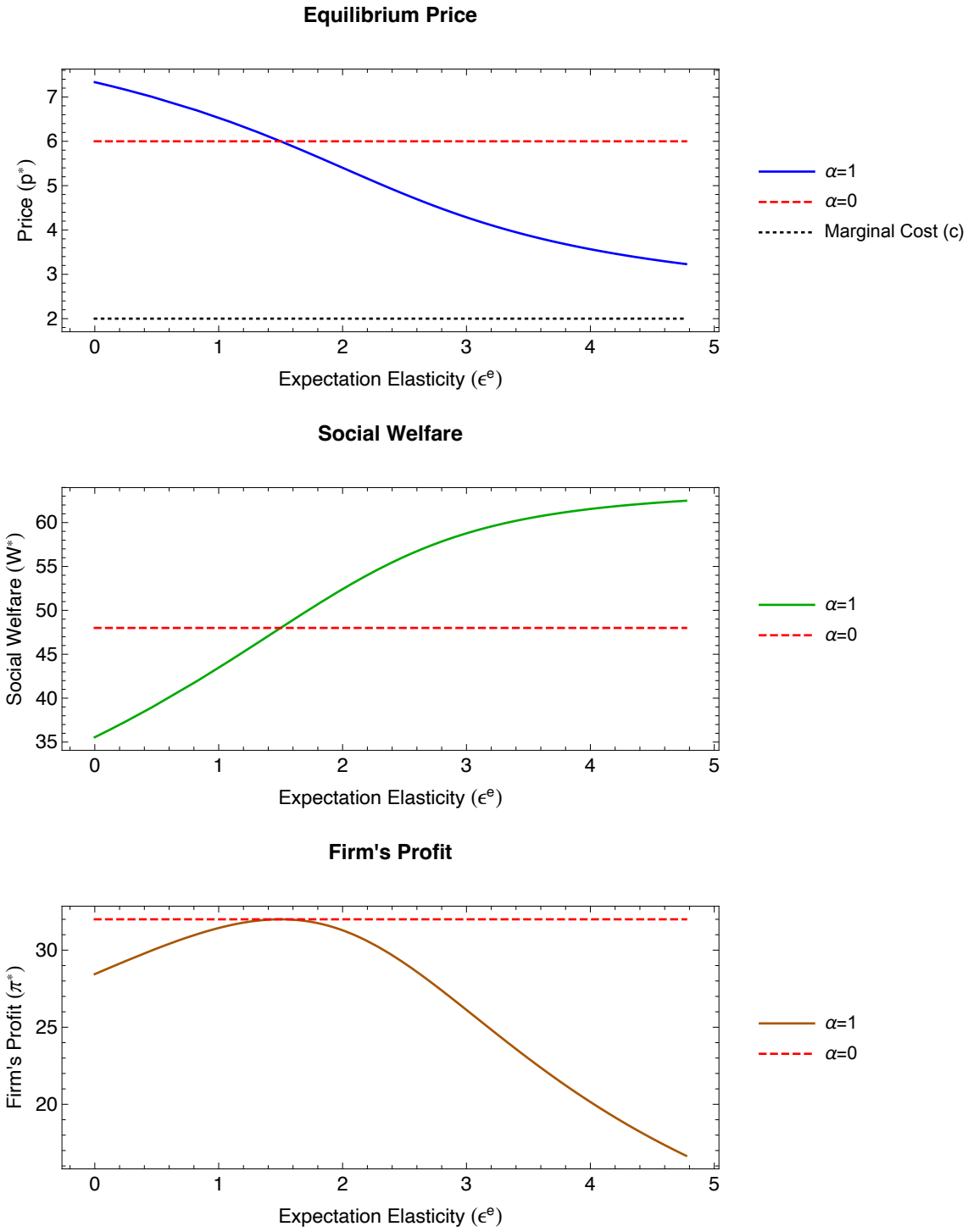


Figure 1: Equilibrium outcomes under monopoly. For the numerical illustration, we set the parameter values as follows: $a = 10$, $b = 1$, $c = 2$, and $v = 0.5$.

4 Extension: Differentiated Duopoly

The previous sections analyzed a monopoly market. In this section, we extend the baseline model to a differentiated duopoly under Bertrand competition, using a representative consumer framework for analytical tractability. This extension is motivated by the prevalence of competition in network industries—multiple firms simultaneously compete for consumers who care about network size (Rochet and Tirole, 2003; Armstrong, 2006). This allows us to examine how strategic interactions—specifically, the cross-expectation effect over network size expectations—interact with the pricing discipline mechanism.

4.1 The General Model

Following Singh and Vives (1984), we consider a market with two firms producing differentiated goods, $i, j = 1, 2$ $(i \neq j)$. The representative consumer ' s preferences are described by a quasi-linear utility function:

$$U(x_1, x_2, y) = V(x_1, x_2) + \sum_{i=1,2} vX_i^e x_i + y,$$

where x_i represents consumption of the numeraire good i, and y is the consumption of a numeraire good. The function $V(x_1, x_2)$ represents the intrinsic (standalone) utility from consuming the differentiated goods, which is assumed to be strictly concave, twice continuously differentiable, and symmetric. The term $vX_i^e x_i$ captures the network externality, where $v \in [0, 1]$ is the intensity of the network effect, and X_i^e is the expected network size of good i.

The representative consumer maximizes utility subject to the budget constraint $\sum_{i=1,2} p_i x_i + y = m$, where p_i is the price of good i, and m is income. The first-order conditions for utility maximization with respect to x_i yield the following inverse demand functions:

$$p_i = \frac{\partial V(x_1, x_2)}{\partial x_i} + vX_i^e.$$

Letting $u_i = vX_i^e - p_i$ be the common net surplus from good i, which excludes the

standalone benefit, we can invert this system to obtain the direct demand functions:

$$x_i = n_i(u_i, u_j), \quad \text{for } i, j = 1, 2, \ i \neq j.$$

We assume that the goods are substitutes ($n_{ij} \equiv \partial n_i / \partial u_j < 0$) and that the own-price effect exceeds the cross-price effect: $n_{ii} \equiv \partial n_i / \partial u_i > 0$ and $n_{ii} > |n_{ij}|$.

Aggregate consumer surplus (CS) is given by total utility minus total expenditure. Substituting the equilibrium quantities x_i^* yields:

$$CS = V(x_1^*, x_2^*) + \sum_{i=1,2} u_i x_i^*.$$

On the supply side, the two firms face an identical constant marginal cost, $c \geq 0$. Each firm i chooses its price p_i simultaneously to maximize its profit: $\pi_i = (p_i - c)x_i$. Social welfare (W) is the sum of consumer surplus and the firms' profits:

$$W = CS + \sum_{i=1,2} \pi_i = V(x_1^*, x_2^*) + \sum_{i=1,2} (vX_i^e - c)x_i^*.$$

In any self-fulfilling equilibrium, the condition $X_i^e = x_i^*$ must hold.

As in the monopoly case, consumers' expected demand for good i is a function of both prices: $X_i^e = \alpha x_i^e(p_i, p_j) + (1 - \alpha)x_i$. Under $\alpha = 1$, expected demand responds to price changes. We denote the absolute sensitivities of expected demand by $e_{ii} \equiv -\partial x_i^e / \partial p_i \geq 0$ and $e_{ji} \equiv \partial x_j^e / \partial p_i \geq 0$. We call e_{ji} the *cross-expectation effect*, representing how a change in firm i's price affects consumer expectations about firm j's network size.

As shown in Appendix C.1, by totally differentiating the demand system yields the actual price sensitivities $\Phi_i \equiv -\partial x_i / \partial p_i$ under each regime. They are given by

$$\Phi_i^{(1)} = n_{ii}(1 + ve_{ii}) - vn_{ij}e_{ji}, \tag{3}$$

$$\Phi_i^{(0)} = \frac{n_{ii}(1 - vn_{jj}) + vn_{ij}n_{ji}}{(1 - vn_{ii})(1 - vn_{jj}) - v^2 n_{ij}n_{ji}}. \tag{4}$$

Throughout this section, we maintain the condition $\Delta \equiv (1 - vn_{ii})(1 - vn_{jj}) - v^2 n_{ij}n_{ji} > 0$, which ensures that $\Phi_i^{(0)} > 0$. To ensure the existence, uniqueness, and stability of the Nash equilibrium in the pricing game, we follow the literature (e.g., Vives, 1999, Section

6.3) and assume the following stability condition holds throughout the analysis:

$$\left|\frac{\partial^2 \pi_i}{\partial p_i^2}\right| > \left|\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\right|.$$

This condition implies that the direct effect of a firm's own price on its marginal profit dominates the cross-price effect generated by its competitor's price; see Appendix D for details.

Comparing these sensitivities, we obtain the duopolistic counterparts of our main propositions.

Proposition 1′. *Given p_j, the price set by firm i under $\alpha = 1$ is strictly lower than that under $\alpha = 0$ if and only if expected demand is sufficiently price-elastic and/or the cross-expectation effect e_{ji} is sufficiently large (i.e., $\Phi_i^{(1)} > \Phi_i^{(0)}$).*

Proposition 2′. *Under $\alpha = 1$, as expected demand becomes more elastic with respect to own price (e_{ii} increases) or the cross-expectation effect becomes stronger (e_{ji} increases), equilibrium prices decrease, leading to an unambiguous improvement in social welfare.*

Proposition 3′. *Under $\alpha = 1$, the equilibrium profit of each firm is an inverted-U function of $\Phi^{(1)}$: it is increasing in e_{ii} and e_{ji} when $\Phi^{(1)} < \Phi^*$ and strictly decreasing when $\Phi^{(1)} > \Phi^*$, where Φ^* denotes the profit-maximizing level of $\Phi^{(1)}$. Unlike the monopoly case—where the $\alpha = 0$ profit always weakly dominates the $\alpha = 1$ profit—the comparison between $\pi^{(0)}$ and the $\alpha = 1$ equilibrium profit is in general ambiguous for the duopoly.*

Corollary 1′. *Under $\alpha = 1$, as consumer expectations become perfectly elastic ($e_{ii} \to \infty$ or $e_{ji} \to \infty$), the duopoly equilibrium prices converge to marginal costs, achieving the first-best allocation.*

Proofs for Propositions 1′, 2′, and 3′ are provided in Appendix C.2. These propositions imply that the "theory of second best" mechanism identified in the monopoly model is significantly amplified in a duopolistic market. The presence of the cross-expectation effect (e_{ji}) introduces a novel disciplinary channel. When consumers hold highly elastic expectations regarding the expansion of a rival's network, any price increase by a firm triggers a severe risk of substantial demand reduction. Consequently, firms reduce their markups to an even greater extent than in the monopoly case. Therefore, elastic ex-

pectations act as a powerful countervailing force that effectively constrains the market power arising from product differentiation, thereby generating a substantial improvement in social welfare (Proposition 2′).

Corollary 1′ explicitly illustrates the theoretical limit of this disciplinary mechanism. As the elasticity of expectations approaches infinity ($\Phi_i^{(1)} \to \infty$), markups are fully eliminated and the market outcome converges precisely to the first-best allocation. In economic terms, perfectly elastic expectations can completely offset the market power inherent in differentiated Bertrand competition, effectively rendering the market perfectly competitive from a welfare perspective.

Furthermore, Proposition 3′ highlights a profound structural shift in the conflict of interest between firms and society when moving from monopoly to duopoly. In the monopoly model, the firm always prefers accurate expectations ($\alpha = 0$). By contrast, under duopolistic competition, accurate expectations intensify pricing game, potentially trapping firms in a prisoner's dilemma. Instead, inaccurate and inelastic expectations (such as passive expectations where $\alpha = 1$ and $e_{ii} = e_{ji} = 0$) serve as an implicit coordination device that softens price competition. Therefore, while society continues to prefer highly elastic expectations to maximize welfare, competing firms now have a strong incentive to induce passive expectations to protect their profit margins. This result demonstrates that competition not only complicates market dynamics but also deepens the conflict of interest between firms and consumers. The precise condition under which this monotonic behavior holds is characterized in Section 4.2.

4.2 An Illustration with Linear Duopolistic Demand

To obtain explicit solutions, we adopt the demand system proposed by Singh and Vives (1984):

$$x_i = A + \beta(vX_i^e - p_i) - \gamma(vX_j^e - p_j), \tag{5}$$

where $A > 0$, $\beta > 0$ is the own-price sensitivity, $\gamma \in (0, \beta)$ is the cross-price sensitivity, and marginal costs are symmetric at c (so $n_{ii} = \beta$, $n_{ij} = -\gamma$). The price sensitivities are

$$\begin{aligned} \Phi^{(1)} &= \beta(1 + ve_{ii}) + v\gamma e_{ji}, \\ \Phi^{(0)} &= \frac{\beta - v(\beta^2 - \gamma^2)}{(1 - v\beta)^2 - v^2\gamma^2}. \end{aligned} \tag{6}$$

Equation (6) implies that the cross-expectation effect (e_{ji}) strictly increases the price sensitivity $\Phi^{(1)}$ because products are substitutes ($\gamma > 0$).

Focusing on a symmetric equilibrium ($p_1 = p_2 = p^*$, $x_1 = x_2 = x^*$) with $X_i^e = x^*$, the explicit equilibrium price is

$$p^* = \frac{c\Phi\{1 - v(\beta - \gamma)\} + A}{\Phi\{1 - v(\beta - \gamma)\} + (\beta - \gamma)}.$$

This solution directly confirms the duopoly counterparts of our general propositions:

- **Proposition 1′:** $p^{(1)} < p^{(0)}$ iff $\Phi^{(1)} > \Phi^{(0)}$, requiring e_{ii} and/or e_{ji} to be sufficiently large. In particular, even a positive cross-expectation effect ($e_{ji} > 0$) alone can discipline pricing; this channel is absent in the monopoly case.

- **Proposition 2′:** An increase in e_{ii} or e_{ji} raises $\Phi^{(1)}$, lowers p^*, and expands output, unambiguously improving social welfare.

- **Proposition 3′:** With $\pi^* = \Phi(p^* - c)^2$, $\delta \equiv \beta - \gamma > 0$, and $B \equiv 1 - v(\beta - \gamma) > 0$, we have $\partial\pi^*/\partial\Phi \propto \delta - \Phi B$, so the profit-maximizing level is $\Phi^* = \delta/B$. At $e_{ii} = e_{ji} = 0$:

$$\Phi^* = \frac{\delta}{B} < \beta = \Phi^{(1)}\big|_{e_{ii}=e_{ji}=0} \quad \Longleftrightarrow \quad \gamma(1 + v\beta) > v\beta^2. \tag{7}$$

  Under condition (7), equilibrium profit is strictly decreasing in e_{ii} and e_{ji} throughout. Moreover, $\Phi^{(0)} < \beta$, confirming that $\pi^*|_{e_{ii}=e_{ji}=0,\,\alpha=1} > \pi^{(0)}$: with passive expectations under $\alpha = 1$, each firm earns strictly more than under $\alpha = 0$. This contrasts sharply with the monopoly case and arises because product market competition under $\alpha = 0$ induces a prisoner's dilemma in pricing, while passive expectations under $\alpha = 1$ restore implicit coordination and allow firms to jointly sustain higher markups.

- **Corollary 1′:** As $e_{ii} \to \infty$ or $e_{ji} \to \infty$, $\Phi^{(1)} \to \infty$ and $p^* \to c$, achieving the first-best allocation.

Figure 2 visualizes the analytical results of the linear duopoly model. The equilibrium price, output per firm, and profit are plotted against $\epsilon^e_{ii} \equiv e_{ii}(p^*_i/x^*_i)$ under parameters satisfying condition (7), so profit is monotonically decreasing throughout. The figure compares the case without the cross-expectation effect ($e_{ji} = 0$, solid lines) against the case with a positive cross-expectation effect ($e_{ji} = 3$, dash-dotted lines). A higher own-expectation elasticity ϵ^e_{ii} consistently drives prices down, increases output, and reduces profits; the cross-expectation effect reinforces all of these patterns.

The bottom panel reveals a contrast with the monopoly case. Under monopoly, the profit curve has an inverted-U shape and bounded above by $\pi^{(0)}$. In the duopoly under condition (7), the profit curve starts strictly *above* $\pi^{(0)}$ at $\epsilon^e_{ii} = 0$ and declines monotonically thereafter. This confirms that passive expectations ($\epsilon^e_{ii} = 0$) soften price competition, enabling both firms to sustain higher markups than the prisoner's dilemma outcome under $\alpha = 0$ would allow. As ϵ^e_{ii} grows, this coordination advantage erodes, and profit eventually falls strictly below $\pi^{(0)}$, confirming Proposition 3′. Thus, the conflict of interest between firms and society in the duopoly has a richer structure than in the monopoly: firms prefer an intermediate level of expectation elasticity (near zero), while society continues to prefer the highest possible elasticity.

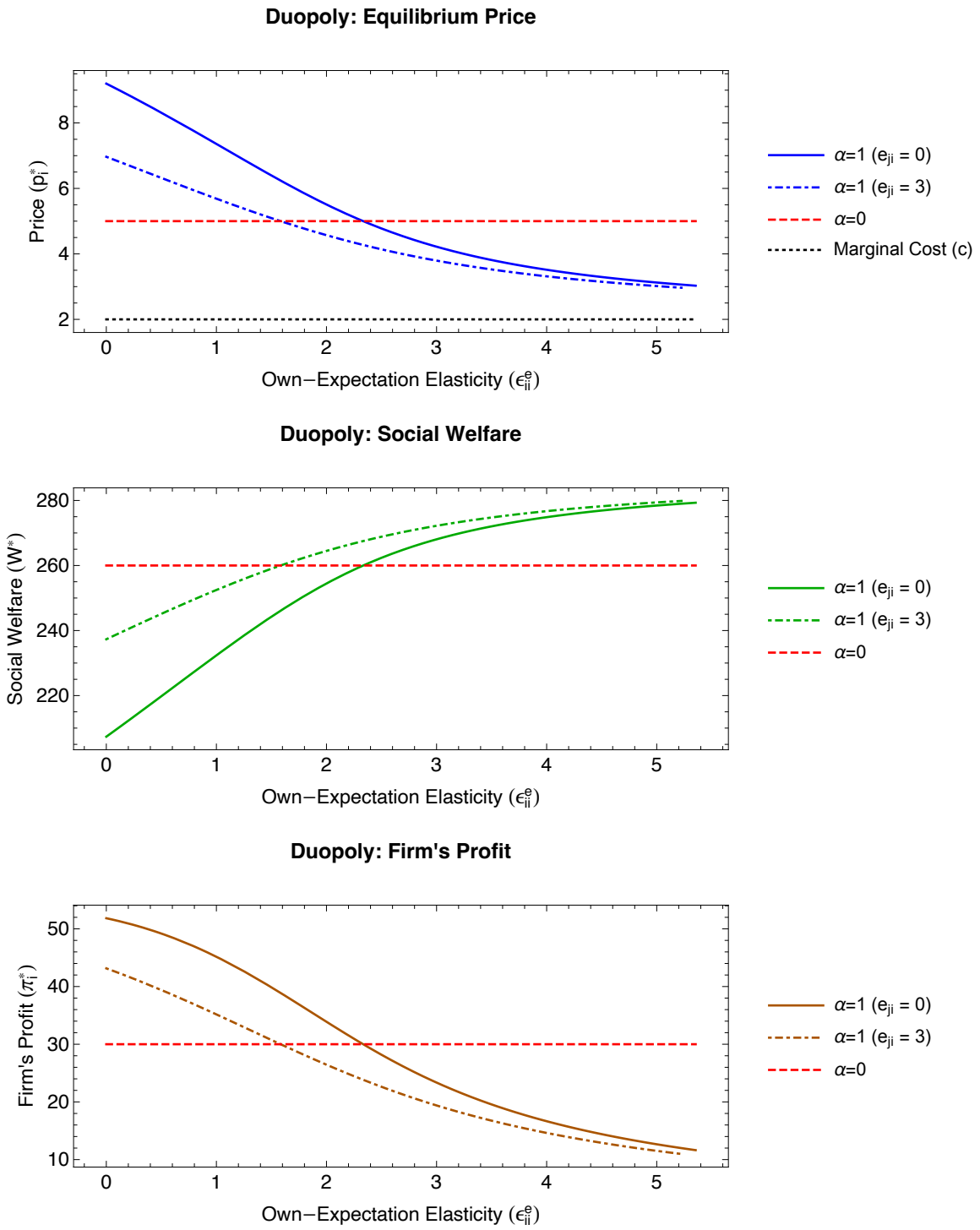


Figure 2: Equilibrium outcomes under duopoly. For the numerical illustration, we set the parameter values as follows: $A = 10$, $\beta = 1$, $\gamma = 0.5$, $c = 2$, and $v = 0.5$.

5 Concluding Remarks

This paper has developed a unified framework that generalizes the conventional dichotomy of passive and responsive consumer expectations in markets with network externalities. Our main finding challenges the standard presumption that accurate expectations are welfare-enhancing: when the price elasticity of expected demand is sufficiently high, the $\alpha = 1$ regime yields lower prices, higher output, and greater social welfare than the responsive expectations benchmark. The driving mechanism follows second-best logic (Lipsey and Lancaster, 1956)—consumers' out-of-equilibrium pessimism about network adoption acts as a countervailing "second distortion" that offsets the monopolist's inherent tendency to underproduce. In the limiting case of perfectly elastic expectations, it fully replicates the competitive outcome (Corollary 1). These results reveal a fundamental conflict of interest: the firm prefers the responsive expectations regime, while society strictly benefits from ever-higher expectation elasticity.

Extending the analysis to a differentiated duopoly, we show that the disciplinary mechanism is reinforced by a cross-expectation effect and that the conflict of interest acquires a richer structure. Unlike the monopoly case, firms under competition may prefer passive expectations to the responsive expectations equilibrium, since inelastic consumer expectations soften price competition and mitigate the prisoner's dilemma in pricing. As expectation elasticity grows, this coordination advantage erodes and profits decline monotonically, while the welfare divergence between firms and society ultimately persists. These results suggest that the appropriate design of consumer-information environments in network industries depends critically on market structure.

Our findings carry a practical policy implication: in network markets dominated by a single firm or a small number of competitors, regulators need not correct consumers' seemingly inaccurate, pessimistic expectations off the equilibrium path. Fostering price-sensitive consumer expectations can serve as an endogenous disciplinary device, alleviating underproduction without direct price regulation. Future research could examine dynamic pricing environments, heterogeneous consumer expectations, and empirical tests of whether the expectation formation mechanisms identified here are detectable in real-world network markets.

Appendix A A Bayesian Microfoundation for $x^e(p)$

This appendix provides a microfoundation for the reduced-form expectation function $x^e(p)$ and establishes when $x^{e\prime}(p) < 0$, following the price-signaling framework of Spence (1973) and Milgrom and Roberts (1986).

Suppose the firm privately observes a viability parameter $\tilde{\theta} \in [\underline{\theta}, \bar{\theta}]$ (drawn from a distribution with density $g(\theta) > 0$), which determines the potential network size via an increasing adoption function $\xi(\theta)$. Consumers observe only the posted price p and apply Bayes' rule to infer $\tilde{\theta}$. In a separating equilibrium with a strictly monotone pricing schedule $p = P(\theta)$, the inference is exact and expected network size is

$$x^e(p) \;=\; \mathbb{E}\Big[\xi(\tilde{\theta}) \mid p\Big] \;=\; \xi\big(P^{-1}(p)\big)\,.$$

Differentiating with respect to p gives

$$x^{e\prime}(p) \;=\; \left.\frac{\xi'(\theta)}{P'(\theta)}\right|_{\theta = P^{-1}(p)}. \tag{8}$$

Since $\xi'(\theta) > 0$, the sign of $x^{e\prime}(p)$ is determined solely by the sign of $P'(\theta)$. In the network externalities context, higher-type firms have an incentive to price *lower* to attract a critical mass of adopters, yielding a downward-sloping schedule ($P'(\theta) < 0$) and hence $x^{e\prime}(p) < 0$: a higher price leads consumers to infer a less viable network, consistent with the 3DO example in Section 1. This grounds the assumption $x^{e\prime}(p) \leq 0$ maintained throughout the paper. Furthermore, Equation (8) provides a structural interpretation of ϵ^e: it measures how informative the price is as a signal of network viability, so a higher ϵ^e corresponds to a steeper equilibrium pricing schedule and stronger disciplinary pressure on the firm.

Appendix B Proof of Proposition 3

In any rational expectations equilibrium, the condition $X^e = x = n(u)$ must hold. Therefore, any equilibrium outcome must satisfy $p = vn(u) - u$, which represents the fulfilled-expectations demand curve. Under $\alpha = 0$, the firm maximizes profit directly along this

curve:

$$\max_u \pi(u) = \max_u \{n(u)\,[vn(u) - u] - C(n(u))\}\,.$$

Let $u^{(0)}$ denote the profit-maximizing common net surplus and $\pi^{(0)} = \pi(u^{(0)})$ the resulting maximum profit. Under $\alpha = 1$, the firm maximizes profit subject to $X^e = x^e(p)$, but the resulting equilibrium must also lie on the fulfilled-expectations demand curve. Since $\pi^{(0)}$ is the global maximum on this curve, $\pi^{(0)} \geq \pi^{(1)}$ for all ϵ^e.

For the second part, we first establish that $p^{(1)}$ is strictly decreasing in ϵ^e. Define the equilibrium condition under $\alpha = 1$ as

$$F(u,\, e) \;\equiv\; n'(u)\big[p - C'(n(u))\big] - \frac{n(u)}{1 + ve} \;=\; 0,$$

where $e \equiv -x^{e'} \geq 0$ is the absolute price sensitivity of expected demand and $p = vn(u) - u$ on the fulfilled-expectations demand curve. Computing the partial derivatives:

$$\frac{\partial F}{\partial e} = \frac{v\,n(u)}{(1 + ve)^2} > 0, \qquad \frac{\partial F}{\partial u} = \pi''(u^*) < 0,$$

where the sign of $\partial F/\partial u$ follows from the second-order condition (Appendix D). By the implicit function theorem,

$$\frac{du^*}{de} = -\frac{\partial F/\partial e}{\partial F/\partial u} = -\frac{v\,n(u)/(1 + ve)^2}{\pi''(u^*)} > 0.$$

Differentiating $p = vn(u) - u$ along the fulfilled-expectations curve then gives

$$\frac{dp^*}{de} = \big(vn'(u) - 1\big)\frac{du^*}{de} < 0,$$

where $vn'(u) < 1$ is the stability condition implicit in Equation (1). Since $\epsilon^e = e \cdot p^*/n(u^*)$ is strictly increasing in e in a neighborhood of equilibrium, it follows that $p^{(1)}$ is strictly decreasing in ϵ^e, so $u^{(1)}$ increases monotonically. The profit $\pi(u^{(1)})$ satisfies $\pi'(u) > 0$ for $u < u^{(0)}$ and $\pi'(u) < 0$ for $u > u^{(0)}$. When $\epsilon^e < \epsilon$, $u^{(1)} < u^{(0)}$ and $\pi(u^{(1)})$ increases as ϵ^e rises toward ϵ. At $\epsilon^e = \epsilon$, $u^{(1)} = u^{(0)}$ and $\pi^{(1)} = \pi^{(0)}$. When $\epsilon^e > \epsilon$, $u^{(1)} > u^{(0)}$ and $\pi(u^{(1)})$ decreases. Therefore, the equilibrium profit under $\alpha = 1$ is an inverted-U function of ϵ^e, attaining its maximum $\pi^{(0)}$ at $\epsilon^e = \epsilon$. □

Appendix C Derivations and Proofs for the Duopoly Model

C.1 Derivation of Equations (3) and (4)

Case $\alpha = 1$: The actual demand is $x_i = n_i(vx_i^e(p_i, p_j) - p_i, vx_j^e(p_i, p_j) - p_j)$. Applying the chain rule:

$$\left.\frac{\partial x_i}{\partial p_i}\right|_{\alpha=1} = n_{ii}(-ve_{ii} - 1) + n_{ij}(ve_{ji}) = -\left[n_{ii}(1 + ve_{ii}) - vn_{ij}e_{ji}\right].$$

Since $\Phi_i \equiv -dx_i/dp_i$, Equation (3) follows immediately.

Case $\alpha = 0$: The actual demands are self-referential: $x_i = n_i(vx_i - p_i, vx_j - p_j)$ and $x_j = n_j(vx_j - p_j, vx_i - p_i)$. Differentiating both equations with respect to p_i and rearranging into matrix form:

$$\begin{pmatrix} 1 - vn_{ii} & -vn_{ij} \\ -vn_{ji} & 1 - vn_{jj} \end{pmatrix} \begin{pmatrix} \frac{\partial x_i}{\partial p_i} \\ \frac{\partial x_j}{\partial p_i} \end{pmatrix} = \begin{pmatrix} -n_{ii} \\ -n_{ji} \end{pmatrix}.$$

Using Cramer's rule, with $\Delta \equiv (1 - vn_{ii})(1 - vn_{jj}) - v^2 n_{ij} n_{ji} > 0$:

$$\frac{\partial x_i}{\partial p_i} = \frac{-n_{ii}(1 - vn_{jj}) - vn_{ij}n_{ji}}{\Delta}.$$

Hence $\Phi_i^{(0)} = -\partial x_i/\partial p_i$ yields Equation (4). □

C.2 Proofs of Propositions 1′, 2′, and 3′

Proof of Proposition 1′. From $p_i = c + x_i/\Phi_i$, the price is strictly decreasing in Φ_i. Thus $p_i^{(1)} < p_i^{(0)}$ iff $\Phi_i^{(1)} > \Phi_i^{(0)}$, which requires

$$n_{ii}(1 + ve_{ii}) - vn_{ij}e_{ji} > \frac{n_{ii}(1 - vn_{jj}) + vn_{ij}n_{ji}}{\Delta}.$$

Since $n_{ij} < 0$ and $e_{ji} \geq 0$, the term $-vn_{ij}e_{ji}$ is nonnegative, and the left-hand side is strictly increasing in e_{ii} and e_{ji}. Sufficiently elastic expected demands ensure that the inequality holds. □

Proof of Proposition 2′. To assess the effect of consumer expectations on social welfare, we first examine the response of the equilibrium price p^*. Let e parameterize the elasticity of expectations, where an increase in e leads to a higher $\Phi^{(1)}$.

The first-order condition for firm i is $\Pi_i(p^*, p^*, e) = 0$. Fully differentiating this with respect to p^* and e yields

$$\frac{dp^*}{de} = -\frac{\frac{\partial \Pi_i}{\partial e}}{\frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial^2 \pi_i}{\partial p_i \partial p_j}}.$$

Regarding the numerator, $\partial \Pi_i / \partial e = -(p^* - c)(\partial \Phi^{(1)} / \partial e) < 0$. This indicates that more elastic expectations exert downward pressure on marginal profit. Regarding the denominator, the stability condition (direct effect dominates cross-effect) ensures that $\partial^2 \pi_i / \partial p_i^2 + \partial^2 \pi_i / \partial p_i \partial p_j < 0$. Consequently, we obtain $dp^*/de < 0$, meaning that the equilibrium price strictly decreases as consumer expectations become more elastic.

Finally, we consider the implications for social welfare, W. In this symmetric equilibrium, social welfare decreases with the equilibrium price ($dW/dp^* < 0$), since lower prices expand total output toward the socially optimal level and mitigate the distortions. Since $dp^*/de < 0$, it follows that

$$\frac{dW}{de} = \frac{dW}{dp^*} \cdot \frac{dp^*}{de} > 0.$$

Thus, more elastic consumer expectations strictly improve social welfare, completing the proof. □

Proof of Proposition 3′. To establish the inverted-U relationship and the condition under which $B(\Phi^{(1)}) < 0$, we apply the implicit function theorem to the symmetric equilibrium. Let $X(p^*) \equiv n(vp^* - p^*, vp^* - p^*)$ denote the actual demand function evaluated at a symmetric fulfilled-expectations equilibrium ($X^e = x^* = X(p^*)$). Differentiating $X(p^*)$ with respect to p^* yields

$$X'(p^*) = -(1 - v)(n_{ii} + n_{ij}).$$

Given the standard assumptions $n_{ii} > |n_{ij}|$ and $v \in [0, 1]$, we have $X'(p^*) \leq 0$.

At a symmetric fulfilled-expectations equilibrium, the equilibrium price p^* must satisfy

the following first-order condition:

$$p^* - c = \frac{X(p^*)}{\Phi^{(1)}}. \tag{9}$$

Since p^* is determined by $\Phi^{(1)}$ through this identity, we can differentiate both sides of (9) with respect to $\Phi^{(1)}$ to perform comparative statics:

$$\frac{dp^*}{d\Phi^{(1)}} = \frac{X'(p^*)\frac{dp^*}{d\Phi^{(1)}}\Phi^{(1)} - X(p^*)}{(\Phi^{(1)})^2}.$$

Solving for the equilibrium price response $dp^*/d\Phi^{(1)}$, we obtain

$$\frac{dp^*}{d\Phi^{(1)}} = -\frac{X(p^*)}{\Phi^{(1)}(\Phi^{(1)} - X'(p^*))}. \tag{10}$$

The equilibrium profit is given by $\pi^* = (p^* - c)^2\Phi^{(1)}$. Differentiating π^* with respect to $\Phi^{(1)}$ yields

$$\frac{d\pi^*}{d\Phi^{(1)}} = (p^* - c)\left[(p^* - c) + 2\Phi^{(1)}\frac{dp^*}{d\Phi^{(1)}}\right] \equiv (p^* - c)B(\Phi^{(1)}).$$

Substituting (9) and (10) into $B(\Phi^{(1)})$, yields

$$B(\Phi^{(1)}) = -X(p^*)\frac{\Phi^{(1)} + X'(p^*)}{\Phi^{(1)}(\Phi^{(1)} - X'(p^*))}.$$

Since $X(p^*) > 0$, $\Phi^{(1)} > 0$, and $\Phi^{(1)} - X'(p^*) > 0$, the sign of $B(\Phi^{(1)})$ depends entirely on the numerator term $-(\Phi^{(1)} + X'(p^*))$. Therefore, we can characterize the profit dynamics as follows:

- If $\Phi^{(1)} < -X'(p^*)$, then $\Phi^{(1)} + X'(p^*) < 0 \implies B(\Phi^{(1)}) > 0$.
- If $\Phi^{(1)} = -X'(p^*)$, then $\Phi^{(1)} + X'(p^*) = 0 \implies B(\Phi^{(1)}) = 0$.
- If $\Phi^{(1)} > -X'(p^*)$, then $\Phi^{(1)} + X'(p^*) > 0 \implies B(\Phi^{(1)}) < 0$.

This confirms that as $\Phi^{(1)}$ increases, the firm's profit initially increases, attains a unique maximum at $\Phi^{(1)} = -X'(p^*)$, and subsequently decreases. This formally establishes the inverted-U structure and rigorously demonstrates that $B < 0$ holds for sufficiently large $\Phi^{(1)}$, completing the proof. □

Conflict of interest. The firm maximizes profit at $\Phi^{(1)} = \Phi^*$, while Proposition 2′ and Corollary 1′ imply social welfare is strictly increasing in $\Phi^{(1)}$ without bound. Accordingly, the firm's preferred elasticity is strictly below the socially optimal level. □

Appendix D Equilibrium Existence, Uniqueness, and Second-Order Conditions

D.1 Monopoly

Second-order condition. The monopolist's problem is to solve $\max_u \pi(u) = n(u)D(u) - C(n(u))$. The first-order condition is given by (2). The second-order condition (SOC) requires

$$\pi''(u) = n''(u)[p - C'(n(u))] + 2n'(u)D'(u) + n(u)D''(u) - [n'(u)]^2 C''(n(u)) < 0. \quad (11)$$

At a first-order optimum, $p - C'(n(u)) = -n(u)D'(u)/n'(u) > 0$, so the first term in (11) is positive if $n''(u) > 0$ and negative if $n''(u) < 0$. The second term, $2n'(u)D'(u)$, is negative (since $n' > 0$ and $D' < 0$). The third term $n(u)D''(u)$ and the fourth term $-[n'(u)]^2 C''(n(u))$ have signs that depend on the curvature of $D(\cdot)$ and $C(\cdot)$, respectively. We impose (11) as an assumption on the primitives throughout the paper. This condition is satisfied under standard regularity conditions (e.g., log-concave demand and weakly convex costs). This includes the linear case analyzed in Section 3.4, where $D'' = 0$, $n'' = 0$, and $C'' = 0$, so the SOC reduces to $2n'(u)D'(u) < 0$, which always holds.

Uniqueness. Under the SOC (11), the profit function $\pi(u)$ has a unique interior maximum u^* over the relevant domain, thereby ensuring that equilibrium price and output are uniquely determined. In the linear demand example, the profit function is strictly concave in u (since $\pi''(u) = 2bD'(u) < 0$ uniformly), guaranteeing global uniqueness.

D.2 Differentiated Duopoly

Stability and existence of Bertrand equilibrium. In the duopoly model of Section 4, we require that the system of best-response functions has a unique fixed point. Following the standard conditions for differentiated Bertrand competition (Singh and Vives, 1984),

we impose

$$\Delta \equiv (1 - vn_{ii})(1 - vn_{jj}) - v^2 n_{ij} n_{ji} > 0, \tag{12}$$

which ensures that $\Phi_i^{(0)} > 0$ and that the $\alpha = 0$ price sensitivities in (4) are well-defined. Condition (12) is equivalent to requiring that the network externalities are not so strong as to cause consumers' network utility to exceed the intrinsic price sensitivity. It is satisfied whenever $vn_{ii} < 1$ and $v^2 n_{ij} n_{ji} < (1 - vn_{ii})(1 - vn_{jj})$.

Under the $\alpha = 1$ regime, $\Phi_i^{(1)} = n_{ii}(1 + ve_{ii}) - vn_{ij}e_{ji} > 0$ by inspection (since $n_{ii} > 0$, $e_{ii} \geq 0$, $n_{ij} < 0$, and $e_{ji} \geq 0$, all terms are nonnegative). The Bertrand equilibrium under $\alpha = 1$ is the unique solution to the symmetric system $p_i = c + x_i/\Phi_i^{(1)}$ for $i = 1, 2$, whose existence and uniqueness follow from the contraction mapping theorem under the standard stability condition that own-price effects dominate cross-price effects, i.e., $|\partial^2 \pi_i / \partial p_i^2| > |\partial^2 \pi_i / \partial p_i \partial p_j|$ at the optimum.

Second-order conditions for each firm. Under the linear demand system (5), firm i's profit is strictly concave in p_i for given p_j under $\alpha = 1$, since $\partial^2 \pi_i / \partial p_i^2 = -2\Phi^{(1)} < 0$. This establishes the SOC and confirms that each firm's first-order condition defines a global maximum.

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