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<tr>
<td><strong>Author</strong></td>
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<tr>
<td><strong>Citation</strong></td>
<td>Osaka Journal of Mathematics. 20(4); 793-801</td>
</tr>
<tr>
<td><strong>Issue Date</strong></td>
<td>1983-12</td>
</tr>
<tr>
<td><strong>ISSN</strong></td>
<td>0030-6126</td>
</tr>
<tr>
<td><strong>Textversion</strong></td>
<td>Publisher</td>
</tr>
<tr>
<td><strong>Relation</strong></td>
<td>The OJM has been digitized through Project Euclid platform <a href="http://projecteuclid.org/ojm">http://projecteuclid.org/ojm</a> starting from Vol. 1, No. 1.</td>
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Placed on: Osaka City University
NEWMAN'S THEOREM FOR PSEUDO-SUBMERSIONS

HSU-TUNG KU, MEI-CHIN KU AND L.N. MANN

(Received June 4, 1982)

1. Introduction. In 1931 M.H.A. Newman [N] proved the following result.

**Theorem** (Newman). *If M is a connected topological manifold with metric d, there exists a number \( \varepsilon = \varepsilon(M, d) > 0 \), depending only upon M and d, such that every finite group G acting effectively on M has at least one orbit of diameter at least \( \varepsilon \).*

P.A. Smith [S] in 1941 proved a version of Newman's Theorem in terms of coverings of \( M \) and Dress [D] in 1969 gave a simplified proof of Newman's Theorem based on Newman's original approach and using a modern version of local degree.

In another direction Cernavskii [C] in 1964 generalized Newman's Theorem to the setting of finite-to-one open mappings on manifolds. His techniques were based upon those of Smith. Recently McAuley and Robinson [M-R] and Deane Montgomery [MO] have expanded upon Cernavskii's results. In fact McAuley and Robinson, using the techniques of Dress, have obtained the following version of Cernavskii's result. [M-R, Theorem 3].

**Theorem** (Cernavskii-McAuley-Robinson). *If M is a compact connected topological manifold with metric d, there exists a number \( \varepsilon = \varepsilon(M, d) > 0 \) such that if \( Y \) is a metric space and \( f: M \to Y \) a continuous finite-to-one proper open surjective mapping which is not a homeomorphism, then there is at least one \( y \in Y \) such that \( \text{diam } f^{-1}(y) \geq \varepsilon \).*

In [H-M] we gave estimates of the \( \varepsilon \) in Newman's Theorem for Riemannian manifolds \( M \) in terms of convexity and curvature invariants of \( M \). In this note we apply the techniques of [H-M] to obtain estimates of \( \varepsilon \) for the Cernavskii-McAuley-Robinson result for the case where \( M \) is a Riemannian manifold. In particular, if \( S^n \) is the standard unit sphere with standard metric, we show \( \varepsilon > \pi/2 \), i.e. if \( f: S^n \to Y \) is as above, there exists \( y \in Y \) with \( \text{diam } f^{-1}(y) > \pi/2 \). We also obtain a cohomology version of Newman's Theorem for compact orientable Riemannian manifolds which generalizes Theorem 3 of
We wish to thank McAuley and Robinson for sending us [M-R] prior to its publication.

We shall call an open finite-to-one proper surjective map \( f: M \to Y \), \( Y \) a metric space, which is not a homeomorphism, a pseudo-submersion, and \( f^{-1}(f(x)) \) an orbit of \( f \) at \( x \) and denoted by \( O_f(x) \).

Now let \( M \) be a connected Riemannian manifold with a metric induced from the Riemannian metric of \( M \). Assume that there exists at least one pseudo-submersion \( f: M \to Y \). Define the Newman's diameter \( d^T(M) \) of \( M \) by

\[
d^T(M) = \sup \left\{ \varepsilon \left| \exists \text{ any pseudo-submersion } f: M \to Y. \quad \exists x \in M \text{ such that } \text{diam } O_f(x) \geq \varepsilon \right. \right\}
\]

Define the cardinality of \( f \) by \( \text{Card } f = \max \{ \text{card } O_f(x) : x \in M \} \). Suppose there exists at least one pseudo-submersion \( f: M \to Y \) with \( \text{Card } f = p > 1 \); we define the mod \( p \) Newman's diameter \( d^T_p(M) \) as the supremum of the numbers \( \varepsilon > 0 \) such that for every pseudo-submersion \( g: M \to Y \) with \( \text{Card } g = p \), there exists an orbit of diameter at least \( \varepsilon \).

We call a subset \( S \) of a Riemannian manifold \( M \) convex if for every pair of points in \( S \) there exists a unique distance measuring geodesic in \( S \) joining them. For \( x \in M \), the radius of convexity of \( M \) at \( x \), which we denote by \( r_x \), is defined as the supremum of the radii of all convex embedded open balls centered at \( x \).

The following result is based upon Lemma 3 in [D] and appears as Theorem 2 in [M-R].

**Proposition 2.1** (Dress-McAuley-Robinson). Let \( U \) be an open, connected, relatively compact subset of \( \mathbb{R}^n \) and \( f: U \to Y \) a pseudo-submersion. Then

\[
D = \max \left\{ \min \{ ||x-y|| : y \in \partial \bar{U} \} : x \in U \right\} 
\]

\[
\leq C = \max \{ \text{diam } O_f(x) : x \in \partial \bar{U} \} .
\]

Here \( ||x-y|| \) is the euclidean norm.

It is well-known that the exponential map locally stretches distances for manifolds of nonpositive curvature. In [H-M] the following analogous result was obtained for manifolds of bounded curvature.

**Proposition 2.2.** Suppose \( K \leq b^2, b > 0 \), (respectively \( K \leq 0 \)) on a Riemannian manifold \( M \) with distance function \( d \). Let \( B_r(z) = \{ y : d(y, z) < r \} \) be a convex embedded ball centered at \( z \) in \( M \). Suppose further that \( r < \pi b^{-1}/2 \) (respectively \( 0 < r < \infty \) when \( K \leq 0 \)). For any \( x, y \in B_r(z) \), if \( \hat{x} = \exp_z^{-1}x \) and \( \hat{y} = \exp_z^{-1}y \), then
\[d(x, y) \geq (2/\pi) ||\dot{x} - \dot{y}||\] respectively \[d(x, y) \geq ||\dot{x} - \dot{y}||\] when \(K \leq 0\). Here \(||\dot{x} - \dot{y}||\) is the euclidean norm in the tangent space \(M_z\).

Using Propositions 2.1 and 2.2 and the techniques of [H-M] we are able to prove the main result of this section.

**Theorem 2.3.** Let

\[\varphi = \sup_{x \in M} r_x.\]

1. If \(K \leq 0\), \(d^T(M) \geq \varphi/2\). In particular if \(\varphi = +\infty\), there exist point inverses of arbitrarily large diameters.

2. If \(K \leq b^2\), and \(a = \min\{\pi/2b, \varphi\}\), \(d^T(M) \geq 2a/(\pi + 2)\).

Proof. Fix any \(z \in M\) and let \(r_z\) the radius of convexity at \(z\). For any \(r > 0\) satisfying

\[r < \begin{cases} r_z & \text{if } K \leq 0 \\ \min\{r_n, \pi b^{-1}/2\} & \text{if } K \leq b^2, \end{cases}\]

and any \(\alpha, \frac{1}{2} \leq \alpha < 1\), suppose that

\((H)\) \(\text{diam } O_f(x) < (1 - \alpha)r\), all \(x \in M\).

Define \(U = f^{-1}[f(B_{a^2}(z))]\). Clearly \(U\) is open. We claim \(U\) is connected. Let \(V\) be a component of \(U\). Now it is known [C], [MO] that \(V\) maps onto \(f(U) = f(B_{a^2}(z))\). Hence, \(V\) intersects \(O_f(z)\). But since

\[\text{diam } O_f(z) < (1 - \alpha)r \leq \alpha r,\]

\[O_f(z) \subseteq B_{a^2}(z).\]

Furthermore by \((H)\),

\[B_{a^2}(z) \subseteq U \subseteq B_r(z).\]

Let \(U_{\perp} = \exp_z^{-1}U\). Then \(U_{\perp}\) is an open and connected subset of \(R^n = M_z\).

It can be verified that

\[U_{\perp} = \exp_z^{-1}f^{-1}[f(B_{a^2}(z))].\]

Consequently we can apply Proposition 2.1 to \(f_{\perp} = f \circ \exp_z: U_{\perp} \to Y\). Now

\[\{\dot{x} \in M_z \mid ||\dot{x}|| \leq \alpha r\} = \exp_z^{-1}B_{a^2}(z) \subseteq U_{\perp}\]

\[\subseteq \exp_z^{-1}B_r(z) = \{\dot{x} \in M_z \mid ||\dot{x}|| \leq r\}\]

The left-hand inclusion implies

\[D = \max\{\min\{||\dot{x} - \dot{y}|| \mid \dot{y} \in \partial U_{\perp} \mid \dot{x} \in U_{\perp}\} \geq \alpha r\} \text{ (Simply let } \dot{x} = 0)\]

Since \(B_r(z)\) is a convex, embedded ball with \(r < \pi b^{-1}/2\) when \(K \leq b^2\) \((r < \infty\) when \(K \leq 0\), we may apply Proposition 2.2. So
\begin{align*}
C &= \text{Max}\{\text{diam } O_r(x) \mid x \in \partial U_r\} \\
&\leq \begin{cases} \\
\pi/2 \text{Max}\{\text{diam } O_r(x) \mid x \in \partial U\} & \text{if } K \leq 0 \\
(1-\alpha)r & \text{if } K \leq b^2 \\
(1-\alpha)\pi r/2 & \text{if } K \leq b^2 \\
\end{cases}
\end{align*}

by (H).

By Proposition 2.1, \( D \leq C \). Consequently

\[ \alpha < \begin{cases} \\
(1-\alpha)r & \text{if } K \leq 0 \\
(1-\alpha)\pi r/2 & \text{if } K \leq b^2 \\
\end{cases} \]

or

\[ \alpha < \begin{cases} \\
1/2 & \text{if } K \leq 0 \\
\pi/(\pi + 2) & \text{if } K \leq b^2 \\
\end{cases} \]

Consequently, (H) is false for

\[ a = \begin{cases} \\
1/2 & \text{if } K \leq 0 \\
\pi/(\pi + 2) & \text{if } K \leq b^2 \\
\end{cases} \]

So there exists an \( x \in M \) with \( \text{diam } O_r(x) \geq r/2 \) if \( K \leq 0 \); \( 2r/(\pi + 2) \) if \( K \leq b^2 \).

It is possible to obtain a version of Theorem 2.3 in terms of injectivity radius. For a complete connected Riemannian manifold \( M \) define the injectivity radius \( \iota(M) \) by

\[ \iota(M) = \sup \{d(x, C(x)) : x \in M\} \]

where \( C(x) \) denotes the cut locus of \( x \).

\textbf{Theorem 2.4.}

(1) If \( K \leq 0 \), \( d^T(M) \geq \iota(M)/2 \).

(2) If \( K \leq b^2 \), \( M \) is compact and \( a = \text{Min} \{\pi/2b, \iota(M)/2\} \), \( d^T(M) \geq 2a/\pi \).

\textbf{3. Estimate of Newman's diameter } \( d^T(S^n) \) \textbf{and related topics.} We use the notion of \textit{degree of a map} defined by Dress [D].

Let \( f: M^n \rightarrow Y \) be a pseudo-submersion. The \textit{branch set} \( B_f \) of \( f \) is defined as \( B_f = \{x \in M : f \text{ is not a local homeomorphism at } x\} \). By [C] or [M-R], \( M - f^{-1}(f(B_f)) \) is a dense open subset of \( M^n \).

\textbf{Lemma 3.1: Newman's Lemma} (Dress [D], McAuley-Robinson [M-R]). Let \( f: M \rightarrow Y \) be a pseudo-submersion, \( X \) a locally compact metric space, \( g: M \rightarrow X \) and \( j: Y \rightarrow X \) be a proper map such that \( g = j \circ f \). Let \( x \in X \) be such that

\[ g^{-1}(x) \cap f^{-1}(f(B_f)) = \emptyset, \]
and \( y \in j^{-1}(x) \). If \( \text{Card} \ f^{-1}(y)=p \), then \( g \) is inessential at \( x \) for \( Z_p \); that is, the degree of \( g \) at \( x \), \( d(g, x) \), is zero (with \( Z_p \) as coefficients).

**Theorem 3.2.** Let \( M \) be a compact connected oriented topological \( n \)-manifold and \( f: M^n \to Y \) be a pseudo-submersion with \( \text{Card} \ O_f(x_0)=p>1 \) for some \( x_0 \in M^{-f^{-1}(f(B_f))} \). Suppose \( \varphi: M \to S^n \) is a map such that the \( \text{deg} \varphi \equiv 0 \) mod \( p \). If we denote \( \varphi(z) \) by \( \bar{z} \), then there exists \( x \in M \) such that the following holds:

\[
\sum_{z \in O_f(x)} \bar{z} = c \bar{x} \quad \text{in } R^{n+1} \quad \text{for some } c \leq 0.
\]

\[
\begin{cases}
\gamma = \pi & \text{if } \text{Card} \ O_f(x) = 2 \\
\sqrt{\gamma o \bar{z}} & \geq 2\pi/3, \quad \text{and } \sqrt{\gamma o \bar{z}} = \sqrt{\gamma o \bar{y}}, \quad \text{if } \text{Card} \ O_f(x) = 3 \quad \text{and}
\quad O_f(x) = \{x, y, z\}.

\quad \geq \pi - \cos^{-1}(1/(p-1)) > \pi/2 & \text{if } \text{Card} \ O_f(x) \geq 4
\end{cases}
\]

for some \( z \in O_f(x) \), where \( \sqrt{\gamma o \bar{z}} \) denotes the angle between \( o \bar{x} \) and \( o \bar{z} \), \( o \in R^{n+1} \) the origin, and \( S^n \) the standard unit sphere in \( R^{n+1} \).

**Proof.** (1) Suppose on the contrary, then \( \sum_{z \in O_f(x)} \bar{z} \equiv 0 \) for all \( x \) in \( M \).

Define a map \( g: M^n \to S^n \) by

\[
g(x) = \frac{\sum_{z \in O_f(x)} \bar{z}}{|\sum_{z \in O_f(x)} \bar{z}|}.
\]

Then for any \( x \in O_f(x) \), \( g(x) = g(x) \). Hence \( g \) induces a map \( j: Y \to S^n \) such that \( g = j \circ f \). It follows from Lemma 3.1 that \( g \) is inessential at \( g(x) \) for \( Z_p \).

On the other hand, by hypothesis there is a well defined homotopy \( H: M \times [0, 1] \to S^n \) between \( \varphi \) and \( g \) defined by

\[
H(x, t) = \frac{t \varphi(x) + (1-t)g(x)}{|t \varphi(x) + (1-t)g(x)|}.
\]

Hence, \( \text{deg} \varphi = \text{deg} g = d(g, g(x)) = 0 \mod p \). This is a contradiction.

(2) For any \( y, z \in O_f(x) \), set \( \theta_{yz}=\sqrt{\gamma o \bar{z}} \). Let \( \langle, \rangle \) be the standard inner product in \( R^{n+1} \). From (1) there exists an element \( x \) in \( M \) such that

\[
\langle \bar{x}, \bar{z} \rangle + \sum_{z \neq x, z \in O_f(x)} \langle \bar{x}, \bar{z} \rangle = c\langle \bar{x}, \bar{x} \rangle
\]

for some \( c \leq 0 \); that is,

\[
(\ast) \sum_{z \neq x, z \in O_f(x)} \cos \theta_{zx} = c - 1 \leq -1
\]

If \( \text{Card} f=2 \), it is easy to see from (\ast) that \( c=0 \), and \( \theta_{zx}=\pi \).

If \( \text{Card} f=3 \), then \( \cos \theta_{xy} + \cos \theta_{yx} = c - 1 \). From (1) we have

\[
|(1-c)\bar{x} + \bar{z}|^2 = |\bar{y}||^2.
\]

Hence \( \cos \theta_{yx} = \cos \theta_{xy} = (c-1)/2 \). That is, \( \theta_{xy} = \theta_{zx} \geq 2\pi/3 \). If \( \text{Card} f=p \geq 4 \), there exists at least one \( z \in O_f(x) \) such that \( \cos \theta_{zx} \leq -1/(p-1) \); that is, \( \theta_{zx} \geq \pi \).
Theorem 3.2 implies the following:

**Corollary 3.3.** (1) \( d_T^S(S^n) = \pi \), i.e., for any pseudo-submersion \( f: S^n \to Y \) with \( \text{Card } f = 2 \), there exists \( x \in S^n \) such that \( f^{-1}(f(x)) = \{x, -x\} \).
(2) \( d^S(S^n) = 2\pi/3 \).
(3) \( (p-1)\pi/p \geq d^T(S^n) \geq \pi - \cos^{-1}(1/(p-1)) > \pi/2 \) if \( p \geq 4 \).
(4) \( 2\pi/3 \geq d^T(S^n) > \pi/2 \).

**Proof.** In [K], the equivariant diameter \( D(M) \) and modulo \( p \) equivariant diameter \( D_p(M) \) have been defined. They are precisely defined by the pseudo-submersions \( \pi: M \to MG \) which are orbit maps of isometric actions of compact Lie groups \( G \) or \( G = \mathbb{Z}_p \) on \( M \) respectively. Hence \( D(M) \geq d^T(M) \) and \( D_p(M) \geq d^T_p(M) \) for some \( p \). But \( D(S^n) = 2\pi/3 \) and \( D_p(S^n) = (p-1)\pi/p \) if \( p \geq 3 \) by [K]. Hence, the result follows from Theorem 3.2 by applying it to the identity map \( S^n \to S^n \).

**Remarks.** (i) The statement (1) extends the following well known result: For any non-trivial involution \( g \) of \( S^n \), there exists \( x \in S^n \) such that \( gx = -x \).
(ii) By using the arguments of Milnor in [MI] we can also show the following: Let \( f: M \to Y \) and \( \bar{f}: \bar{M} \to \bar{Y} \) be pseudo-submersions with Card \( f = \text{Card } \bar{f} = 2 \), \( B_f = B_{\bar{f}} = \varnothing \), where \( M \) is a compact connected oriented \( n \)-manifold and \( \bar{M} \) a mod 2 homology \( n \)-sphere. Suppose there exists a map \( \varphi: M \to \bar{M} \) of odd degree. Then there exists \( x \in M \) such that \( \varphi \) \( O_f(x) = O_{\bar{f}}(\varphi x) \).

**Theorem 3.4.** Let \( M \) be a compact connected \( n \)-dimensional submanifold of \( R^{n+1} \), \( n \geq 2 \), and let \( y \in R^{n+1} - M \) be in a bounded component. Suppose \( f: M \to Y \) is a pseudo-submersion. Then there exists \( x \in M \) such that
(1) If Card \( f = 2 \), \( \{O_f(x), y\} \) lies on a line in \( R^{n+1} \).
(2) If Card \( f = 3 \), and \( O_f(x) = \{x, u, v\} \), then
\[
\angle xyu = \angle uvy = \angle vyx = 2\pi/3.
\]
In particular \( \{O_f(x), y\} \) lies in a 2-plane in \( R^{n+1} \).
(3) If Card \( f = p \geq 4 \), then \( \angle uvy \geq \pi - \cos^{-1}(1/(p-1)) > \pi/2 \) for some \( u, v \in O_f(x) \), and \( \{O_f(x), y\} \subset R^{p-1} \cap M \), for some \( (p-1) \)-plane \( R^{p-1} \) of \( R^{n+1} \) (if \( n \geq p-2 \) passing through the origin.

**Proof.** Apply Theorem 3.2 to the map \( \varphi: M \to S^n \) defined by \( \varphi(x) = (y-x)/||y-x|| \) because deg \( \varphi = \pm 1 \). The equality in (2) follows from Corollary 3.3 (2).

4. Cohomology version of Newman’s theorem for pseudo-submersions

Let \( f: M \to Y \) be a pseudo-submersion. A subset \( A \) of \( M \) is called satur-
ated if \( A = O_f(A) \), where \( O_f(A) = \cup \{ O_f(x) : x \in A \} \), or equivalently \( A = f^{-1}(f(A)) \). Then there exists an open neighborhood \( V \) of \( x \) which is homeomorphic to \( R^n \) and \( f \mid V : V \to f(V) \) is a homeomorphism. Hence by excision we have

\[ H_n(Y, Y - f(x); Z_p) \approx H_n(f(V), f(V) - f(x); Z_p) \approx Z_p, \]

where \( p = \text{Card } O_f(x) \).

We shall say a pseudo-submersion \( f : M \to Y \) satisfies the (LOA) (local orientable condition for \( A \)) if \( A \) is a closed saturated subset of \( M \), \( B = f(A) \) is closed in \( Y \) and such that the inclusion \( i_B : (Y, B) \to (Y, Y - x) \) induces an isomorphism

\[ i_B^* : H_n(Y, B; Z_p) \to H_n(Y, Y - f(x); Z_p) \]

for some \( x \in M - f^{-1}(f(B_f)) \), Card \( O_f(x) = p \).

The following result extends the cohomology version of Newman's Theorem for group actions [B], [S] due to Smith.

**Theorem 4.1.** Let \( A \) be a closed subspace of a compact oriented \( n \)-manifold \( M \) such that \( H_n(M, A; Z_p) \approx Z_p \). Let \( \mathcal{U} \) be any open covering of \( M \) such that

\[ H^n(K(\mathcal{U}), K(\mathcal{U} \mid A); Z_p) \to H^n(M, A; Z_p) \]

is surjective, where \( K(\mathcal{U}) \) denotes the nerve of the covering \( \mathcal{U} \). Then there does not exist a pseudo-submersion \( f : M \to Y \) satisfying (LOA) and such that each orbit of \( f \) is contained in some open set in \( \mathcal{U} \).

Proof. Suppose the conclusion is false. Then there exists a pseudo-submersion \( f : M \to Y \) satisfying (LOA) and each orbit \( O_f(x) \) is contained in a saturated open set \( V_x \) which is contained in some member of \( \mathcal{U} \). Let \( \mathcal{C} \mathcal{U} = \{ f(V_x) : x \in V \} \). Then \( f^{-1}(\mathcal{C} \mathcal{U}) \) is a refinement of \( \mathcal{U} \). By [B, p. 154], \( f^* : H^n(Y, B; Z_p) \to H^n(M, A; Z_p) \) is an epimorphism. But the Kronecker product induces a canonical epimorphism [G, p. 132]

\[ \alpha : H^n(M, A; Z_p) \to H_n(M, A; Z_p)^* = \text{Hom}(H_n(M, A; Z_p); Z_p); \]

hence we have an isomorphism \( f_* : H_n(M, A; Z_p) \to H_n(Y, B; Z_p) \).

Let \( K = O_f(x) \), and \( O_k \subseteq H_n(M, M - K; Z_p) \) be the fundamental class which is the element such that for any \( z \in K \), the inclusion \( i_z : (M, M - K) \to (M, M - z) \) satisfies \( i_z^*(O_k) = 1, \) the identity element of \( H_n(M, M - z; Z_p) \approx Z_p \) (cf. [D]).

We have the following commutative diagram

\[
\begin{array}{ccc}
Z_p = H_n(M; Z_p) & \xrightarrow{i_*} & H_n(M, A; Z_p) \\
\downarrow & & \downarrow \\
H_n(M, M - K; Z_p) & \xrightarrow{i_z^*} & H_n(M, M - z; Z_p) = Z_p
\end{array}
\]
where all homomorphisms are induced by inclusions. Since \( k_\ast \) is an isomorphism for all \( z \) in \( K \), there exists an element \( a \) in \( H_n(M, A; Z_p) \) such that \( i_\ast(a) = O_K \). Now we consider the following commutative diagram

\[
\begin{array}{ccc}
H_n(M, A; Z_p) & \xrightarrow{f_\ast} & H_n(Y, B; Z_p) \\
\downarrow i_\ast & \approx & \downarrow i_\ast
\end{array}
\]

By definition, \( d(f, f(x)) = f_\ast(O_K) \) (cf. [D]). It follows that

\[
d(f, f(x)) = f_\ast i_\ast(a) = i_\ast f_\ast(a) \neq 0 .
\]

On the other hand, we can apply Lemma 3.1 to the map \( f \), with \( f = j \circ f \), to obtain \( d(f, f(x)) = 0 \), where \( j \) is the identity map. This is an obvious contradiction and the proof of the theorem is complete.

**Corollary 4.2.** Let \( M \) be a compact connected oriented \( n \)-manifold, and \( U \) an open covering of \( M \) such that

\[
(*) \quad H^q(|\sigma|; Z_p) = 0 \quad \text{for any } \sigma \in K(U) \text{ and any } q \geq 1 .
\]

Then there does not exist a pseudo-submersion \( f: M \to Y \) such that

1. \( i_\ast: H_n(Y; Z_p) \cong H_n(Y, Y-x; Z_p) \), where \( i_\ast: Y \to (Y, Y-x) \) is inclusion, \( x \in M - f^{-1}(f(B_\varepsilon)) \), Card \( O_f(x) = p \), and
2. Each orbit of \( f \) is contained in some member of \( U \).

**Proof.** The hypothesis (*) implies that

\[
H^q(K(U); Z_p) \cong H^q(M; Z_p)
\]

for all \( q \geq 0 \) by Leray's Theorem [G-R, p. 189].

As an example, if \( M \) is a compact connected oriented Riemannian manifold, and \( U \) consists of all open convex proper subsets of \( M \), then the condition (*) of Corollary 4.2 is satisfied.

**References**


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