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Abstract

This paper investigates should an airline offer vertically differentiated services, which are substitutes of its own services. The airline operates a certain number of direct flights to offer various types of services including nonstop, one-stop, or multiple-stop services. Homogenous passengers care about the fare and the flight(s) schedule when using a service. Under this general setting, we show that it is optimal for the airline to offer only one type of service in any particular city-pair market. This result supports a number of previous works that primarily argue network efficiency under the condition that only one type of service is offered in a particular market. This result also provides a theoretical explanation for the empirical finding that airlines that offer one-stop service through a hub are less likely to enter the same market with nonstop service than those that do not. This paper also presents an example of that if passengers horizontally differentiate among the type of services by other factors, the airline may offer multiple types of substitutive services in a market on its network.

Keywords: Airline network; Vertically differentiated services; Corner solutions; Schedule competition
JEL classification codes: L13; L93

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1 Introduction

Deregulation started in North America and sequentially in Europe and other regions has led significant changes in the airline industry. In deregulated airline markets, carriers are free to expand and reorganize their route structure, offer various types of services and set prices. This freedom has led a dramatic growth of hub-spoke networks. The economic efficiency of hub-spoke networks generates from economies of traffic density on cost side and/or from flight frequencies effects on demand side.[1] This is known as a positive "hub-spoke network effect" and it becomes a fixing knowledge in the industry.

However, since 1990s, it can be often observed that some major carriers (e.g., US Airways, Continental and Delta) began offering competing nonstop service in markets they already serviced with one-stop service through a hub (for the corresponding markets, see Dunn (2008)).[2] Doubtlessly, this way of offering vertically differentiated services, which are substitutes of the carries' own services, will weaken the described hub-spoke network effects. Therefore, it is an important managerial issue to investigate this airline strategy.

The purpose of this paper is to investigate should an airline offer vertically differentiated services, which are substitutes of its own services. The airline operates a certain number of direct flights to offer various types of services including nonstop, one-stop, or multiple-stop services. Homogenous passengers care about the fare and the flight(s) schedule when using a service. Under this general setting, we show that it is optimal for the airline to offer only one type of service in any particular city-pair market.

This theoretical result is consistent with one of the empirical results of Dunn (2008). Dunn indicated the real-world phenomenon that some major network carriers began offering nonstop service in markets in which it also operates non-

[1] For the studies on cost side, see empirical studies by Caves et al. (1984) and Brueckner and Spiller (1994); theoretical studies by Brueckner and Spiller (1991) and Hendricks et al. (1995). For those on demand side, see Brueckner (2004) and Kawasaki (2008), among others.

[2] A similar example in the Asia-Pacific is that Qantas currently offers one-stop service on the Tokyo-Cairns routes connected in Sydney, while its low cost division, Jetstar offers a substitutive nonstop service in this city-pair market.

stop service through a hub, and found the evidence that airlines that offer one-stop service through a hub are less likely to enter that same market with nonstop service than those that do not. As such, our present result may provide a theoretical explanation for the empirical finding of Dunn (2008).

Furthermore, this theoretical result also supports a number of previous works that primarily argued the efficiency between hub-spoke and fully-connected networks, under the condition that only one type of service is offered in a particular market. In other words, the condition that a carrier does not offer vertically differentiated substitutes in its own market (see Bittlingmayer (1990), Hendricks et al. (1995), Pels et al. (2000), Oum, Zhang and Zhang (1995), Brueckner (2004), Flores-Fillol (2009) among others). Since the setting of this present paper is quite general, our result is widely applicable for calculating airlines' optimal actions (the profit maximization in monopoly and the derivation of best response in oligopoly).

Finally, this paper also considers a plausible case where passengers horizontally differentiate among the type of services by other factors (e.g.,). Then, it is shown that airlines may offer multiple types of substitutive services in a market on its network if the degree of product differentiation is certain large. This finding seems consistent with the results of Kawasaki (2008) and Brueckner and Pai (2009), where differentiation among passengers may persuade an airline to offer both nonstop and one-stop service in a particular market.

The remainder of this article is organized as follows. Section 2 provides an example that a carrier supplies only one service among competing services; Section 3 presents the network structures and basic assumptions; Section 4 shows the main result that a carrier supplies only one service among competing services; Section 5 shows that a carrier supplies all services if competing services are sufficiently not substitute; Section 6 concludes the paper.

2 Example

There are three cities 0, 1 and 2. Let $C := \{0, 1, 2\}$, which is the set of cities. An airline operates three flights between each city pair. For any distinct $i, j \in C$, let ij denote the flight between cities i and j. Let $\mathcal{F} := \{01, 02, 12\}$, which is the set of flights. The airline supplies three nonstop services between each city pair and a one-stop service between cities 1 and 2 with transit at city 0. For any distinct $i, j \in C$, let ij and 102 denote the nonstop service between cities i and j and the one-stop service, respectively. Let $\mathcal{S} := \{01, 02, 12, 102\}$, which is the set of services. Suppose that services 12 and 102 are substitutes for passengers traveling between cities 1 and 2.

For any $i \in \{01, 02\}$, let $p_i(q_i, f_i)$ be the price of nonstop service i when the quantity of service i is q_i and the frequency of flight i is f_i. Let $p_{12}(Q, f_{12})$ be the price of nonstop service 12 when the sum of the quantities of services 12 and 102 is Q and the frequency of flight 12 is f_{12}. Let $p_{102}(Q, f_{01}, f_{02})$ be the price of one-stop service 102 when the sum of the quantities of services 12 and 102 is Q and the frequencies of flights 01 and 02 are f_{01} and f_{02}, respectively. For any $i \in \mathcal{F}$, let $C_i(f_i)$ be the cost of flight i when the frequency of flight i is f_i. For simplicity, the costs based on quantities of services are assumed to be constant and normalized to be zero.[3] Then, the airline's profit with its action $(q, f) = \left((q_i)_{i\in\mathcal{S}}, (f_i)_{i\in\mathcal{S}}\right)$ is

$$\begin{aligned}\pi(q, f) = & p_{01}(q_{01}, f_{01})\, q_{01} + p_{02}(q_{02}, f_{02})\, q_{02} \\ & + p_{12}(q_{12} + q_{102}, f_{12})\, q_{12} + p_{102}(q_{102} + q_{12}, f_{01}, f_{02})\, q_{102} \\ & - C_{01}(f_{01}) - C_{02}(f_{02}) - C_{12}(f_{12}).\end{aligned}$$

Assumption 1 states that in the market between cities 1 and 2, there exists a positive aggregate quantity such that the price of nonstop service 12 is positive.

Assumption 1. For any $f_{12} \in \mathbb{R}_+$, there exists $Q \in \mathbb{R}_{++}$ such that $p_{12}(Q, f_{12}) > 0$.

Assumption 2 states that the profit from nonstop service 12 is greater under

[3] This assumption is not crucial for our results.

some positive frequency of flight 12 than under zero frequency.

Assumption 2. For any $(q_{12}, Q) \in \mathbb{R}_+^2$ such that $0 < q_{12} \leq Q$ and $p_{12}(Q, 0) > 0$, there exists $f_{12} \in \mathbb{R}_{++}$ such that $p_{12}(Q, f_{12}) q_{12} - C_{12}(f_{12}) > p_{12}(Q, 0) q_{12} - C_{12}(0)$.

Assumption 3 states that the cost of flight 12 is greater under any positive frequency than under zero frequency.

Assumption 3. For any $f_{12} \in \mathbb{R}_{++}$, $C_{12}(f_{12}) > C_{12}(0)$.

The following Proposition 1 states that under Assumptions 1–3, it is necessary for the profit maximization not to supply both nonstop service 12 and one-stop service 102 with positive quantities. Before the formal proof, we provide the intuitive reason first. Suppose that under a profit maximizer, the airline supplies services 12 and 102 with positive quantities q_{12}^* and q_{102}^*, respectively. Under this supposition, it will be shown that the prices of services 12 and 102 are equal and positive and the frequency of flight 12 is positive. Let the airline's action change from the maximizer to an alternative action that is the same as the maximizer except that the quantities of services 12 and 102 change to 0 and $q_{12}^* + q_{102}^*$, respectively, and the frequency of flight 12 changes to 0. Then, the aggregate quantity in the market between cities 1 and 2 does not change. Thus, the price of service 102 does not change. Hence, the revenue from service 102 under the alternative action is the sum of revenues from services 12 and 102 under the maximizer; the revenue from service 12 under the alternative action is zero. Therefore, the total revenue does not change. On the other hand, since the frequency of flight 12 decreases from a positive number to zero, the cost of flight 12 decreases. Thus, the profit increases, which contradicts the definition of maximizer.

Proposition 1. *Suppose that Assumptions 1–3 hold. Let* $(q^*, f^*) = \left(\left(q_i^*\right)_{i \in \mathcal{S}}, \left(f_i^*\right)_{i \in \mathcal{F}}\right)$ *be a profit maximizer. Then,* $q_{12}^* = 0$ *or* $q_{102}^* = 0$.

Proof. Suppose that $q_{12}^* > 0$ and $q_{102}^* > 0$. Let $p_{12}^* := p_{12}\left(q_{12}^* + q_{102}^*, f_{12}^*\right)$ and $p_{102}^* := p_{102}\left(q_{102}^* + q_{12}^*, f_{01}^*, f_{02}^*\right)$.

Lemma 1. $p_{12}^* = p_{102}^*$.

Proof. Suppose that $p^*_{12} > p^*_{102}$. Define $(q, f) = \left((q_i)_{i\in\mathcal{S}}, (f_i)_{i\in\mathcal{F}}\right)$ as $q_{12} = q^*_{12} + q^*_{102}$, $q_{102} = 0$, $q_{01} = q^*_{01}$, $q_{02} = q^*_{02}$ and $f = f^*$. Then, $\pi(q, f) - \pi(q^*, f^*) = \left(p^*_{12} - p^*_{102}\right) q^*_{102} > 0$, which is a contradiction. Similarly, $p^*_{12} < p^*_{102}$ leads to a contradiction. Q.E.D.

Lemma 2. $p^*_{12} > 0$.

Proof. Suppose that $p^*_{12} = 0$. By Assumption 1, there exists $Q \in \mathbb{R}_{++}$ such that $p_{12}\left(Q, f^*_{12}\right) > 0$. Define $(q, f) = \left((q_i)_{i\in\mathcal{S}}, (f_i)_{i\in\mathcal{F}}\right)$ as $q_{12} = Q$, $q_{102} = 0$, $q_{01} = q^*_{01}$, $q_{02} = q^*_{02}$ and $f = f^*$. Note that by Lemma 1, $p^*_{102} = 0$. Then, $\pi(q, f) - \pi(q^*, f^*) = p_{12}(Q, f^*) Q > 0$, which is a contradiction. Q.E.D.

Lemma 3. $f^*_{12} > 0$.

Proof. Suppose that $f^*_{12} = 0$. Then, by Lemma 2, $p_{12}\left(q^*_{12} + q^*_{102}, 0\right) > 0$. Thus, by Assumption 2, there exists $f'_{12} > 0$ such that $p_{12}\left(q^*_{12} + q^*_{102}, f'_{12}\right) q^*_{12} - C_{12}\left(f'_{12}\right) > p_{12}\left(q^*_{12} + q^*_{102}, 0\right) q^*_{12} - C_{12}(0)$. Define $(q, f) = \left((q_i)_{i\in\mathcal{S}}, (f_i)_{i\in\mathcal{F}}\right)$ as $f_{12} = f'_{12}$, $f_{01} = f^*_{01}$, $f_{02} = f^*_{02}$, $f_{102} = f^*_{102}$ and $q = q^*$. Then, $\pi(q, f) - \pi(q^*, f^*) = \left(p_{12}\left(q^*_{12} + q^*_{102}, f'_{12}\right) q^*_{12} - C_{12}\left(f'_{12}\right)\right) - \left(p_{12}\left(q^*_{12} + q^*_{102}, 0\right) q^*_{12} - C_{12}(0)\right) > 0$, which is a contradiction. Q.E.D.

Define $(q, f) = \left((q_i)_{i\in\mathcal{S}}, (f_i)_{i\in\mathcal{F}}\right)$ as $q_{12} = 0$, $q_{102} = q^*_{12} + q^*_{102}$, $q_{01} = q^*_{01}$, $q_{02} = q^*_{02}$, $f_{01} = f^*_{01}$, $f_{02} = f^*_{02}$ and $f_{12} = 0$. Then, by Lemma 1, $\pi(q, f) - \pi(q^*, f^*) = C_{12}\left(f^*_{12}\right) - C_{12}(0)$. Thus, by Lemma 3 and Assumption 3, $\pi(q, f) - \pi(q^*, f^*) > 0$, which is a contradiction. Q.E.D.

3 Model

For any sets X and Y, let Y^X be the set of functions from X to Y. For any $f \in Y^X$, for any $x \in X$, let $f_x := f(x)$. For any $f \in Z^{X\times Y}$, for any $x \in X$ and any $y \in Y$, let $f_{xy} := f((x, y))$. For any $f \in Y^X$, for any $(S, g) \in 2^X \times Y^X$, let $f|^g_S$ be the function from X to Y such that for any $x \in S$, $f|^g_S(x) = g(x)$ and for any $x \in X \setminus S$, $f|^g_S(x) = f(x)$: $f|^g_S$ is the function obtained by replacing the values of f with those of g on S. For any $f \in Y^X$, for any $(S, y) \in 2^X \times Y$, let $f|^y_S := f|^g_S$, where g is the function from X to Y such that $g(X) = \{y\}$: $f|^y_S$ is the function obtained by replacing the values of f with

y on S. For any $f \in Y^X$, for any $(x, y) \in X \times Y$, let $f|_x^y := f|_{\{x\}}^y$: $f|_x^y$ is the function obtained by replacing the value of f at x with y.

Let C be a finite set. We call a member of C a *city*.

Definition 1. A *flight* is a set of two distinct members in C.

For example, flight $\{i, j\}$ represents the flight between cities i and j.

Definition 2. A *service* is a set i of some flights such that there exists a injection $j : \{0, 1, \ldots, K\} \to C$ with $K \in \mathbb{Z}_{++}$ such that $i = \{\{j_k, j_{k+1}\} \mid k \in \{0, 1, \ldots, K-1\}\}$.

We call j_0 and j_K *terminal cities* of service i. For example, service $\{\{i, j\}, \{j, k\}\}$ represents the service between cities i and k with transit at city j.

Let $(\mathcal{S}, \mathcal{F})$ be a pair such that $\mathcal{S}$ is a set of some services, $\mathcal{F}$ is a set of some flights, and $\mathcal{F} = \bigcup \mathcal{S}$. Any service in $\mathcal{S}$ uses some flight in $\mathcal{F}$. Any flight in $\mathcal{F}$ is used by some service in $\mathcal{S}$. Define an equivalence relation $\sim$ on $\mathcal{S}$ as for any $i, j \in \mathcal{S}$, $i \sim j$ if and only if the set of terminal cities of i is equal to the set of terminal cities of j. For example, $\{\{i, k\}\} \sim \{\{i, j\}, \{j, k\}\}$. Let $[i] := \{j \in \mathcal{S} \mid j \sim i\}$.

Example 1. If for some $h \in C$, $\mathcal{F} = \left\{i \in 2^C \mid |i| = 2 \wedge i \ni h\right\}$ and $\mathcal{S} = \left\{i \in 2^{\mathcal{F}} \mid |i| \in \{1, 2\}\right\}$, $(C, \mathcal{F}, \mathcal{S})$ represents a hub-spoke network (city h is the hub city).

Example 2. If for some $h \in C$, $\mathcal{F} = \left\{i \in 2^C \mid |i| = 2\right\}$ and $\mathcal{S} = \left\{i \in 2^{\mathcal{F}} \mid |i| = 1\right\} \cup \left\{i \in 2^{\mathcal{F}} \mid |i| = 2 \wedge \forall j \in i, j \ni h\right\}$, $(C, \mathcal{F}, \mathcal{S})$ represents a hub-spoke network with non-stop services between non-hub cities.

For any $\alpha \in [0, 1]^{\mathcal{S}^2}$, for any $(i, j) \in \mathcal{S}^2$, α_{ij} represents a degree of substitution between services i and j. For any $i \in \mathcal{S}$, let p_i be a function from $\mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}} \times [0, 1]^{\mathcal{S}^2}$ to $\mathbb{R}_+$ such that for any $(q, f, \alpha), (q', f', \alpha') \in \mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}} \times [0, 1]^{\mathcal{S}^2}$, if for any $j \in [i]$, $q_j = q'_j$, for any $j \in i$, $f_j = f'_j$ and for any $j \in [i]$, $\alpha_{ij} = \alpha'_{ij}$, then, $p_i((q, f, \alpha)) = p_i((q', f', \alpha'))$. $p_i((q, f, \alpha))$ represents the price of service i when the quantity of each service $j \in \mathcal{S}$ is q_j and the frequency of each flight $j \in \mathcal{F}$ is f_j. The price of service i depends only on the quantities of services that compete with service i (services in $[i]$), frequencies of flights that are used in service i (flights in i) and degrees of substitution between service i and services that compete with service i. Let

$C : \mathbb{R}_+^{\mathcal{F}} \to \mathbb{R}_+$. $C(f)$ represents the cost for flights when the frequency of each flight $i \in \mathcal{F}$ is f_i. Define $\pi : \mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}} \times [0,1]^{\mathcal{S}^2} \to \mathbb{R}$ as for any $(q, f, \alpha) \in \mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}} \times [0,1]^{\mathcal{S}^2}$, $\pi((q, f, \alpha)) = \sum_{i \in \mathcal{S}} p_i((q, f, \alpha)) q_i - C(f)$. The profit is the sum of revenues from services minus the cost of flights.

Assumption 4 means that the price of each service depends only on the aggregate quantity in the market in which the service is supplied and the frequencies of flights used in the services.

Assumption 4. For any $i \in \mathcal{S}$, for any $\alpha \in [0,1]^{\mathcal{S}^2}$, for any $(q, f), (q', f') \in \mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}}$, if $\sum_{j \in [i]} \alpha_{ij} q_j = \sum_{j \in [i]} \alpha_{ij} q'_j$ and for any $j \in i$, $f_j = f'_j$, then, $p_i((q, f, \alpha)) = p_i((q', f', \alpha))$.

Assumption 5 ensures the positive revenues in each service.

Assumption 5. For any $i \in \mathcal{S}$, for any $\alpha \in [0,1]^{\mathcal{S}^2}$, for any $f \in \mathbb{R}_+^{\mathcal{F}}$, for some $q \in \mathbb{R}_+^{\mathcal{S}}$ such that $q_i > 0$, $p_i((q, f, \alpha)) > 0$.

Assumption 6 states that the price of a service is increasing in the frequency of each flight used in the service.

Assumption 6. For any $i \in \mathcal{S}$, for any $(q', f', \alpha') \in \mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}} \times [0,1]^{\mathcal{S}^2}$ such that $p_i((q', f', \alpha')) > 0$, if p_i is partially differentiable with respect to f_j at (q', f', α'), then, $\frac{\partial p_i}{\partial f_j}((q', f', \alpha')) > 0$.

Assumption 7 states that if the revenue from a service is positive when the frequency of a flight used by the service is zero, the profit increases as the flight frequency slightly increases from zero.

Assumption 7. For any $(i, j) \in \mathcal{S} \times \mathcal{F}$, for any $(q', f', \alpha') \in \mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}} \times [0,1]^{\mathcal{S}^2}$, if π is partially differentiable with respect to f_j at $\left(q', f'|_j^0, \alpha'\right)$, if $p_i\left(\left(q', f'|_j^0, \alpha'\right)\right) q'_i > 0$, then, $\frac{\partial \pi}{\partial f_j}\left(\left(q', f'|_j^0, \alpha'\right)\right) > 0$.

Assumptions 8 and 9 state that the inverse demand functions and the cost function are differentiable in flight frequencies.

Assumption 8. For any $i \in \mathcal{S}$, for any $(q', f', \alpha') \in \mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}} \times [0,1]^{\mathcal{S}^2}$ such that $p_i((q', f', \alpha')) > 0$, p_i is partially differentiable with respect to f_j at (q', f').

Assumption 9. C is partially differentiable.

In this paper, we regard $\mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}}$, $[0,1]^{\mathcal{S}^2}$ as the subspace of the $|\mathcal{S}| + |\mathcal{F}|$-dimensional and $|\mathcal{S}|^2$-dimensional Euclid spaces, respectively. Assumption 10 states that the profit is a continuous function.

Assumption 10. π is continuous.

Assumption 11 implies that there exists a compact subset of $\mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}}$ such that under any degree of substitution, any profit maximizer is in this compact subset. The profit maximization with capacity constraints for service quantities and flight frequencies is equivalent to the profit maximization under Assumption 11 without the capacity constraints.

Assumption 11. There exists a compact subset C of $\mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}}$ such that for any $\alpha \in [0,1]^{\mathcal{S}^2}$, for any $(q,f) \in \mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}} \setminus C$, for some $(q',f') \in C$, $\pi((q,f,\alpha)) < \pi((q',f',\alpha))$.

Example 3. If for some $T \in \left[0, \frac{1}{\max\{|i| | i \in \mathcal{S}\}}\right)$, for any $i \in \mathcal{S}$, for any $(q,f,\alpha) \in \mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}} \times [0,1]^{\mathcal{S}^2}$, $p_i((q,f,\alpha)) = \max\left\{1 - T|i| + \frac{\sum_{j \in i} f_j}{|i|} - \sum_{j \in [i]} \alpha_{ij} q_j, 0\right\}$ and for any $f \in \mathcal{F}$, $C(f) = \sum_{i \in \mathcal{F}} f_i^2$, Assumptions 4–10 are satisfied.

Let M be a correspondence from $[0,1]^{\mathcal{S}^2}$ to $\mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}}$ such that for any $\alpha \in [0,1]^{\mathcal{S}^2}$, $M(\alpha) = \arg\max_{(q,f) \in \mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}}} \pi((q,f,\alpha))$.

4 Profit maximizers under perfectly substitutive demands

Proposition 2 is the main result of this paper. This proposition states that if vertically differentiated competing services (i.e., services between the same pair of terminal cities) are perfectly substitutive,[4] it is necessary for the profit maximization not to supply multiple competing services with positive quantities.

[4] Consider Example 3. Suppose that for any $i, j \in \mathcal{S}$ such that $i \sim j$, $\alpha_{ij} = 1$. Then, competing services are perfectly substitutive. On the other hand, due to T and f, competing services are vertically differentiated.

Proposition 2. *Suppose that Assumptions 4–9 hold. Let* $\alpha \in [0,1]^{\mathcal{S}^2}$. *Let* $(q^*, f^*) \in M(\alpha)$. *If for any* $i, j \in \mathcal{S}$ *such that* $i \sim j$, $\alpha_{ij} = 1$, *then, for any* $i, j \in \mathcal{S}$ *such that* $i \neq j$ *and* $i \sim j$, $q_i^* = 0$ *or* $q_j^* = 0$.

Proof. Suppose that Assumptions 4–9 hold. Let $\alpha \in [0,1]^{\mathcal{S}^2}$. Let $a^* = (q^*, f^*) \in M(\alpha)$. For any $i \in \mathcal{S}$, let $p_i^* := p_i((q^*, f^*, \alpha))$. Suppose that for any $i, j \in \mathcal{S}$ such that $i \sim j$, $\alpha_{ij} = 1$. In the following, since α is fixed, we omit α from the arguments of p_i and π.

Lemma 4. *For any* $i, j \in \mathcal{S}$ *such that* $i \neq j$ *and* $i \sim j$, *if* $q_i^* > 0$ *and* $q_j^* > 0$, *then,* $p_i^* = p_j^*$.

Proof. Suppose that $p_i^* > p_j^*$. Then, by Assumption 4, $\pi\left(\left(q^*|_i^{q_i^*+q_j^*}|_j^0, f^*\right)\right) - \pi(a^*) = \left(p_i^* - p_j^*\right) q_j^* > 0$, which is a contradiction. Similarly, $p_i^* < p_j^*$ leads to a contradiction. Q.E.D.

Lemma 5. *For any* $i \in \mathcal{S}$, *if* $q_i^* > 0$, *then,* $p_i^* > 0$.

Proof. Suppose that $p_i^* = 0$. By Assumption 5, for some $q \in \mathbb{R}_+^{\mathcal{S}}$ such that $q_i > 0$, $p_i((q, f^*)) > 0$. Thus, $p_i\left(\left(q^*|_{[i]}^q, f^*\right)\right) = p_i((q, f^*)) > 0$. Note that by Lemma 4, for any $j \in [i]$ such that $q_j^* > 0$, $p_j^* = p_i^* = 0$, and thus, for any $j \in [i]$, $p_j^* q_j^* = 0$. Then, $\pi\left(\left(q^*|_{[i]}^q, f^*\right)\right) - \pi(a^*) = \sum_{j \in [i]} p_j\left(\left(q^*|_{[i]}^q, f^*\right)\right) q_j \geq p_i\left(\left(q^*|_{[i]}^q, f^*\right)\right) q_i > 0$, which is a contradiction. Q.E.D.

Lemma 6. *Let* q' *be a function from* $\mathcal{S}$ *to* $\mathbb{R}_+$ *such that for any* $i \in \mathcal{S}$, $\sum_{j \in [i]} q_j' = \sum_{j \in [i]} q_j^*$ *and if* $q_i^* = 0$, $q_i' = 0$. *Then, for any* $(i, j) \in \mathcal{S} \times \mathcal{F}$ *such that* $j \in i$, $p_i((q, f)) q_i$ *is partially differentiable with respect to* f_j *at* (q', f^*).

Proof. If $q_i' = 0$, the conclusion is obviously obtained. Suppose that $q_i' > 0$. Then, by the definition of q', $q_i^* > 0$. Thus, by Lemma 5, $p_i^* > 0$. Hence, by $\sum_{j \in [i]} q_j' = \sum_{j \in [i]} q_j^*$ and Assumption 4, $p_i((q', f^*)) = p_i^* > 0$. Therefore, by Assumption 8, the conclusion is obtained. Q.E.D.

Lemma 7. *For any* $(i, j) \in \mathcal{S} \times \mathcal{F}$ *such that* $j \in i$, *if* $q_i^* > 0$, *then,* $f_j^* > 0$.

Proof. Suppose that $f_j^* = 0$. By Lemma 6 and Assumption 9, π is partially differentiable with respect to f_j at (q^*, f^*). By $q_i^* > 0$ and Lemma 5, $p_i((q^*, f^*))\,q_i^* > 0$. Note that by $f_j^* = 0$, $(q^*, f^*) = \left(q^*, f^*|_j^0\right)$. Then, by Assumption 7, $\frac{\partial \pi}{\partial f_j}((q^*, f^*)) > 0$. Hence, for some $f_j \in \mathbb{R}_{++}$, $\pi\left(\left(q^*, f^*|_j^{f_j}\right)\right) > \pi(a^*)$, which is a contradiction. Q.E.D.

Let $i, j \in \mathcal{S}$. Suppose that $i \neq j$ and $i \sim j$. Suppose that $q_i^* > 0$ and $q_j^* > 0$. Without loss of generality, since $i \neq j$, there exists $k \in \mathcal{F}$ such that $k \in i$ and $k \notin j$. Let $a' = (q', f') := \left(q^*|_i^0|_j^{q_i^*+q_j^*}, f^*\right)$. By Lemma 6 and Assumption 9, for any $l \in \mathcal{S}$, $p_l(q, f)\,q_l$ and π are partially differentiable with respect to f_k at a^* and a'. Note that since $q_i' = 0$, $\frac{\partial p_i(q,f) q_i}{\partial f_k}((q', f')) = 0$. Note also that since $k \notin j$, $\frac{\partial p_j(q,f) q_j}{\partial f_k}((q^*, f^*)) = 0$ and $\frac{\partial p_j(q,f) q_j}{\partial f_k}((q', f')) = 0$. Note also that by Assumption 4, for any $l \in \mathcal{S} \setminus \{i, j\}$, for any $f_k \in \mathbb{R}_{++}$, $p_l\left(q^*, f^*|_k^{f_k}\right) q_l^* = p_l\left(q', f'|_k^{f_k}\right) q_l'$, and thus, $\frac{\partial p_l(q,f) q_l}{\partial f_k}((q^*, f^*)) = \frac{\partial p_l(q,f) q_l}{\partial f_k}((q', f'))$. Then, $\frac{\partial \pi}{\partial f_k}(a^*) - \frac{\partial \pi}{\partial f_k}(a') = \frac{\partial p_i((q,f)) q_i}{\partial f_k}(a^*)$. Note that by $q_i^* > 0$ and Lemma 5, $p_i^* > 0$, and thus, by Assumption 8, p_i is partially differentiable with respect to f_k at a^*. Then, $\frac{\partial \pi}{\partial f_k}(a^*) - \frac{\partial \pi}{\partial f_k}(a') = \frac{\partial p_i}{\partial f_k}(a^*)\,q_i^*$. Note that by $p_i^* > 0$ and Assumptions 6 and 8, $\frac{\partial p_i}{\partial f_k}(a^*) > 0$. Note also that $q_i^* > 0$. Then, $\frac{\partial \pi}{\partial f_k}(a^*) - \frac{\partial \pi}{\partial f_k}(a') > 0$. Note that since $a^* \in M$ and by $q_i^* > 0$ and Lemma 7, $f_k^* > 0$, $\frac{\partial \pi}{\partial f_k}(a^*) = 0$. Then, $\frac{\partial \pi}{\partial f_k}(a') < 0$. Note that $a' = \left(q^*|_i^0|_j^{q_i^*+q_j^*}, f^*\right)$ and $f_k^* > 0$. Then, there exists $a'' = \left(q^*|_i^0|_j^{q_i^*+q_j^*}, f^*|_k^{f_k}\right)$ for some $f_k \in \mathbb{R}_+$ such that $\pi(a'') > \pi(a')$. Note that by the construction of a' and Assumption 4, $\pi(a^*) - \pi(a') = \left(p_i^* - p_j^*\right) q_i^*$, and thus, by $q_i^* > 0$ and $q_j^* > 0$, and Lemma 4, $\pi(a^*) = \pi(a')$. Then, $\pi(a^*) < \pi(a'')$, which is a contradiction. Q.E.D.

5 Profit maximizers under sufficiently not substitutive demands

Proposition 3 states that if vertically differentiated competing services (i.e., services between the same pair of terminal cities) are not substitutes, it is necessary for the profit maximization to supply all services with positive quantities.

Proposition 3. *Suppose that Assumptions 4 and 5 hold. Let* $\alpha \in [0,1]^{\mathcal{S}^2}$. *Let* $(q^*, f^*) \in M(\alpha)$. *If for any* $i, j \in \mathcal{S}$ *such that* $i \sim j$, *if* $i = j$, $\alpha_{ij} = 1$ *and if* $i \neq j$, $\alpha_{ij} = 0$, *then, for any* $i \in \mathcal{S}$, $q_i^* > 0$.

Proof. Suppose that Assumptions 4 and 5 hold. Let $\alpha \in [0,1]^{\mathcal{S}^2}$. Let $(q^*, f^*) \in M(\alpha)$. Suppose that for any $i, j \in \mathcal{S}$ such that $i \sim j$, if $i = j$, $\alpha_{ij} = 1$ and if $i \neq j$, $\alpha_{ij} = 0$ (♡). Suppose that for some $i \in \mathcal{S}$, $q_i^* = 0$. By Assumption 5, for some $q \in \mathbb{R}_+^{\mathcal{S}}$ such that $q_i > 0$, $p_i((q, f^*, \alpha)) > 0$. Let $q' := q^*|_i^{q_i}$. Then, by Assumption 4 and supposition (♡), $\pi((q', f^*, \alpha)) - \pi((q^*, f^*, \alpha)) = p_i((q', f^*, \alpha)) q_i' - p_i((q^*, f^*, \alpha)) q_i^*$. Note that $q_i^* = 0$; by the construction of q', Assumption 4 and supposition (♡), $p_i((q', f^*, \alpha)) q_i' = p_i((q, f^*, \alpha)) q_i$. Then, $\pi((q', f^*, \alpha)) - \pi((q^*, f^*, \alpha)) = p_i((q, f^*, \alpha)) q_i > 0$, which is a contradiction. Q.E.D.

Proposition 4 states that if competing services are sufficiently not substitutive, it is necessary for the profit maximization to supply all services with positive quantities.

Proposition 4. *Suppose that Assumptions 4, 5, 10 and 11 hold. Let* α *be a function* $\mathcal{S}^2$ *to* $[0,1]$ *such that for any* $i, j \in \mathcal{S}$ *such that* $i \sim j$, *if* $i = j$, $\alpha_{ij} = 1$ *and if* $i \neq j$, $\alpha_{ij} = 0$. *If* $M(\alpha)$ *is finite, then, there exists neighborhood* N *of* α *such that for any* $\alpha' \in N$, *for any* $(q^*, f^*) \in M(\alpha')$, *for any* $i \in \mathcal{S}$, $q_i^* > 0$.

Proof. Suppose that Assumptions 4, 5, 10 and 11 hold. Let α be a function $\mathcal{S}^2$ to $[0,1]$ such that for any $i, j \in \mathcal{S}$ such that $i \sim j$, if $i = j$, $\alpha_{ij} = 1$ and if $i \neq j$, $\alpha_{ij} = 0$. Suppose that $M(\alpha)$ is finite. By Assumption 11, there exists a compact subset C of $\mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}}$ such that for any $\alpha \in [0,1]^{\mathcal{S}^2}$, for any $\mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}} \setminus C$, for some $(q, f) \in C$, $\pi((q, f, \alpha)) < \pi((q', f', \alpha))$. Let D be the correspondence from $[0,1]^{\mathcal{S}^2}$ to $\mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}}$ such that for any $\alpha \in [0,1]^{\mathcal{S}^2}$, $D(\alpha) = C$. Then, for any $\alpha \in [0,1]^{\mathcal{S}^2}$, $M(\alpha) \subset C = D(\alpha)$. Thus, for any $\alpha \in [0,1]^{\mathcal{S}^2}$, $M(\alpha) = \arg\max_{(q,f) \in D(\alpha)} \pi((q, f, \alpha))$. Note that by Assumption 10, π is a continuous function, and by the difinition of D, D is a continuous compact-valued correspondence. Then, the Berge's maximum theorem, M is a upper-hemicontinuous correspondence. Let $\bar{q}$ be the member of $\mathbb{R}_+^{\mathcal{S}}$ such that for any $i \in \mathcal{S}$, $\bar{q}_i = \min_{(q,f) \in M(\alpha)} q_i$. Note that since $M(\alpha)$ is finite, $\bar{q}$ is well defined and by Proposition 3, for any $i \in \mathcal{S}$, $\bar{q}_i > 0$. Let $S := \left\{(q, f) \in \mathbb{R}_+^{\mathcal{S}} \times \mathbb{R}_+^{\mathcal{F}} \mid \forall i \in \mathcal{S}, q_i > \frac{\bar{q}_i}{2}\right\}$. Then, S is an open set, $M(\alpha) \subset S$, and for any $(q, f) \in S$, for any $i \in \mathcal{S}$, $q_i > 0$. Since M is a upper-hemicontinuous correspondence and S is an open set, there exists a neighborhood N of α such that for any $\alpha' \in N$, $M(\alpha') \subset S$. Therefore, for any $\alpha' \in N$, for any $(q^*, f^*) \in M(\alpha')$, $(q^*, f^*) \in S$, and thus, by the definition of S, for any $i \in \mathcal{S}$, $q_i^* > 0$. Q.E.D.

6 Conclusion

This paper has been motivated based on the common observation that some major carriers tended to enter markets with nonstop services in which they have already offered substitutive one-stop services through their hubs. Under a quite general setting, we have formally demonstrated that this way of offering vertically differentiated services cannot lead major carriers to obtain the largest profit as long as the services are perfectly substitutive. This demonstration provides clear managerial implications for airlines strategies.

However, if competing services in a particular market are horizontally differentiated, i.e., are not perfectly substitutive due to other factors rather than the number of stops, then, offering multiple types of competing services may be a reasonable strategy for the carriers. In the real world, instead of the nonstop entry by themselves, some major carriers have entered the market by establishing a low-cost nonstop division to offer differentiated services. This paper has shown that if competing services are sufficiently not substitutive, offering all services with positive quantities is optimal for an airline. This finding seems consistent with the studies of Kawasaki (2008) and Brueckner and Pai (2009), who showed that differentiation among passengers may persuade an airline to offer both nonstop and one-stop services in a particular market.

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