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Summary

The purpose of this paper is to extend a single-bounded dichotomous choice linear projection model (Watanabe and Asano, forthcoming) to a double-bounded dichotomous choice model. With a dichotomous choice linear projection model, it is possible to make a consistent estimation of mean willingness to pay only through a sample average weighted by the density function of a bid. It is, however, not possible to apply this model to a double-bounded model, since the density in the second stage is unknown. This paper extends this model to a double-bounded case without impairing the simplicity of the single-bounded linear projection model. In addition, this paper shows that like the single-bounded linear projection model, the double-bounded linear projection model does not contain any risk of a specification error. It also indicates that the efficiency obtained in the double-bounded linear projection model is almost equivalent to that of a parametric model based on the Monte Carlo simulation.

Key words: contingent valuation, double-bounded dichotomous choice, mean willingness to pay, linear projection, consistent estimation

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1. Introduction

The purpose of this paper is to extend the linear projection model in a single-bounded dichotomous choice contingent valuation (CV) developed by Watanabe and Asano (forthcoming) to a double-bounded dichotomous choice case. In many practical applications, a parametric model typified by Hanemann et al. (1991) is used for the double-bounded dichotomous choice CV. It is possible to make an efficient estimation from the parametric model by adopting a maximum likelihood method only when the probability distribution is correctly specified. This efficiency, however, is always accompanied with inconsistency caused by specification error. In order to avoid such inconsistency, Carson et al. (1994) and McFadden (1994) applied the self-consistency algorithm (Turnbull, 1974, 1976) to a double-bounded dichotomous choice CV. With this nonparametric model based on Turnbull, it is not possible to explicitly obtain any estimator of the survival function of willingness to pay (WTP). This requires a numerical calculation similar to the EM algorithm. For this reason, there is a limitation on bringing it into practical application[2]. In addition, only the lower bound of the mean WTP is consistently estimated, because the consistency of the survival function is ensured only at each bid point.

On the other hand, with a single-bounded dichotomous choice linear projection model, it is possible to make a consistent estimation of the mean WTP only through a sample average weighted by the density function of a bid. Moreover, it is characterized by the fact that there is no risk of specification error. Considering that a double-bounded dichotomous choice case is frequently employed in practical application, a simple estimation model needs to be developed. This is required not only from the viewpoint of practical operational possibility but also from the viewpoint of the elimination of arbitrariness of investigators for estimation as well as the improvement of evaluation transparency. One of the main objectives of applying CV is to make a cost-benefit analysis of public projects. Thus, ensuring the reliability of evaluation is an essential issue.

[1] See Day (2007) and Haab and McConnell (1997).

As mentioned above, with a linear projection model, it is possible to make a consistent estimation of the mean WTP by calculating a sample average weighted by the density function of a bid. As will be described later, however, the model cannot be applied to a double-bounded case without making any modification because an answer in the second stage is dependent on an answer in the first stage. For this reason, in this paper, a method is developed to extend a linear projection model to the double-bounded dichotomous choice CV. In order to provide a simplified explanation, this paper explains only a double-bounded case, although the model can also be applied to a multiple-bounded case.

This paper consists of the following sections: In Section 2, existing estimation models in the double-bounded dichotomous choice CV are overviewed in order to clarify the features of the model in this paper. In Section 3, the model developed by Watanabe and Asano (forthcoming) is briefly reviewed and problems regarding the extension to a double-bounded case are described. In Section 4, a linear projection model for double-bounded dichotomous choice is developed. In Section 5, through the Monte Carlo simulation, performances in a small sample are examined. Finally, in Section 6, the conclusion and future development are explained.

2. Existing estimation models in double-bounded dichotomous choice CV

To clarify the features of a linear projection model, we will first provide an overview of a typical estimation model, which has frequently been used in the double-bounded dichotomous choice CV.

In a standard double-bounded dichotomous choice model, WTP of observation i, WTP_i, is specified with the following exponential type:

$$WTP_i = exp(z_i\theta + \eta_i) \cdots (1)$$

Here, z_i is the attribute vector of i, θ is the corresponding parameter vector, and η_i is the random disturbance term with independent identical distribution. In addition, the cumulative distribution function of the random disturbance term is $G(t) \equiv Pr(\eta_i \leq t)$. The probability that a respondent i says "pay (yes)" both at the first stage ($k = 1$) and at the second stage ($k = 2$) when a bid, $t_{ki}(k = 1,2, i = 1,2,\cdots,n)$, is offered for improvement of certain environmental quality is $Pr(y_i, y_i)$; the probability that the respondent says "yes" at the first stage and "do not pay (no)" is $Pr(y_i, n_i)$; the probability that the respondent says "no" at the first stage and "yes" at the second stage is $Pr(n_i, y_i)$; and the probability that the respondent says "no" both at the first and the second stage is $Pr(n_i, n_i)$. At this point, the probabilities are as follows:

$$Pr(y_i, y_i) = 1 - G(ln(t_{2i}) - z_i\theta) \quad , \quad Pr(y_i, n_i) = G(ln(t_{2i}) - z_i\theta) - G(ln(t_{1i}) - z_i\theta) \quad ,$$
$$Pr(n_i, y_i) = G(ln(t_{1i}) - z_i\theta) - G(ln(t_{2i}) - z_i\theta), \quad Pr(n_i, n_i) = G(ln(t_{1i}) - z_i\theta) \quad \cdots (2)$$

If the indicator function is defined as follows, then the log likelihood shown below is obtained: If a respondent i says "yes" both at the first and second stage, $I_i^{yy} = 1$, and otherwise $I_i^{yy} = 0$. If the respondent says "yes" at the first stage and "no" at the second stage, $I_i^{yn} = 1$, and otherwise $I_i^{yn} = 0$. If the respondent says "no" at the first stage and "yes" at the second stage, $I_i^{ny} = 1$, and otherwise I_i^{ny} =0. If the respondent says "no" both at the first and second stage, $I_i^{nn} = 1$, and otherwise $I_i^{nn} = 0$.

$$lnL(\theta|I,z,t) = \sum_{i=1}^{n} I_i^{yy}\, ln(Pr(y_i, y_i)) + I_i^{yn}\, ln(Pr(y_i, n_i)) + I_i^{ny}\, ln(Pr(n_i, y_i))$$
$$+ I_i^{nn}\, ln(Pr(n_i, n_i)) \cdots (3)$$

ifying some probability distributions for the random disturbance term and maximizing equation (3), the maximum likelihood estimate, $\hat{G}$, is obtained; then the calculation of $\int \left(1 - \hat{G}\right)dt$ provides the mean WTP, since $1 - G(\cdot)$ is the survival function of WTP. In order to make a consistent estimation of the survival function of WTP, however, it is necessary to correctly specify the WTP function of equation (1) and the probability distribution of the random disturbance term. Nevertheless, it is difficult to correctly specify the WTP function and probability distribution of the random disturbance term because we cannot know them in advance. For this reason, there is no guarantee that a consistent estimation of the survival function can be made; therefore, there is a risk of estimating the mean WTP in error. To avoid this specification error, Carson et al. (1994) and McFadden (1994) applied the self-consistency algorithm (Turnbull, 1974, 1976) to a double-bounded dichotomous choice CV, to estimate the survival function of WTP in a non-parametrical model. This non-parametric estimator, however, cannot be solved explicitly and it requires cumbersome numerical calculation. Thus, as mentioned in Section 1, the practical application of this model is limited. In addition, a consistent estimation is made only at each bid point. Hence, this estimator is nothing more than the lower bound consistent estimator of the mean WTP.

As mentioned above, the method adopted to estimate the mean WTP in the existing model is to estimate the survival function of WTP and then integrate it. Watanabe and Asano (forthcoming) instead developed a model to make a consistent estimation of the mean WTP itself without estimating the survival function of WTP. This point significantly differentiates the linear projection model from the existing models. This paper attempts to extend this idea to double-bounded dichotomous choice CV.

3. Linear projection model of single-bounded dichotomous choice CV

3.1 Mean WTP

A bid for certain environmental quality improvement at the kth stage is defined as t_k. The indicator function is defined so that $y_k = 1$ if the respondent says “pay (yes),” and $y_k = 0$ if the respondent says “do not pay (no)” when t_k is provided. The survival function of the WTP, which is distributed in the range of $[0, B]$, is defined as $S(t) \equiv Pr(WTP \geq t)$. At end points, $S(0) \equiv 1$ and $S(B) \equiv 0$ are assumed. Here, from its definition, y_k is the Bernoulli random variable; hence, the conditional expectation of y_k with respect to t_k is $E(y_k|t_k) = Pr(y_k = 1|t_k)$. In addition, $y_k = 1$ for bid t_k is equivalent to $WTP \geq t_k$ and hence, $Pr(y_k = 1|t_k) = Pr(WTP \geq t_k) = S(t_k)$. The following equation is thus obtained.

$$E(y_k|t_k) = S(t_k) \cdots (4)$$

From equation (4), the mean WTP is calculated as a conditional expectation of y with respect to t as shown below:

$$E(WTP) = \int_0^B S(t)dt = \int_0^B E(y|t)dt \cdots (5)$$

3.2 Estimation model

In the single-bounded linear projection model, first consider the following linear projection of $y_1/f_1(t_1)$ on $x_1 \equiv (1, t_1)'$:[3]

$$LP\left(\frac{y_1}{f_1(t_1)}|x_1\right) = x_1^{'}\beta_1 \Leftrightarrow \frac{y_1}{f_1(t_1)} = x_1^{'}\beta_1 + v_1, E(x_1 v_1) = 0 \cdots (6)$$

Here, $\beta_1 \equiv (\beta_{11}, \beta_{12})'$ is the projection parameter vector, v_1 is the projection error, and $f_1(t_1)$ is the density function of bid t_1. In addition, it is assumed that bid t_1 follows continuous distribution in $[0, B]$, and $f_1(t_1) \neq 0$ in the range. By changing equation (6) we obtain $E(y_1|t_1) = x_1^{'}\beta_1 f_1(t_1) + E(v_1|t_1)f_1(t_1)$ and $E(WTP)$ is calculated as the following:[4]

$$E(WTP) = \int_0^B (\beta_{11} + \beta_{12}t)f_1(t)dt + \int_0^B E(v_1|t)f_1(t)dt = \int_0^B (\beta_{11} + \beta_{12}t)f_1(t)dt \cdots (7)$$

In addition, since $\int_0^B (\beta_{11} + \beta_{12}t)f(t)dt = E(x_1^{'}\beta_1) = E\left(\frac{y_1}{f_1(t_1)}\right)$, the mean WTP can be also represented in the following equation:

$$E(WTP) = E\left(\frac{y_1}{f_1(t_1)}\right) \cdots (8)$$

Estimators for equation (7) and equation (8) are as follows:

$$\hat{E}(WTP) = \int_0^B (\hat{\beta}_{11} + \hat{\beta}_{12}t)f_1(t)dt \cdots (9)$$

$$\hat{E}(WTP) = n^{-1}\sum_{i=1}^{n} \frac{y_{1i}}{f_1(t_{1i})} \cdots (10)$$

Here, $\hat{\beta}_1 \equiv (\hat{\beta}_{11}, \hat{\beta}_{12})'$ is a consistent estimate of β_1. This consistent estimate can be calculated

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[2] For a linear projection, see Wooldridge (2002) pp. 24–27.

[4] From the definition of linear project, $\int_0^B E(v_1|t)f_1(t)dt = E(v_1) = 0$.

by ordinary least squares (OLS), which regresses $y_1/f_1(t_1)$ on x_1, because, from the definition of β_1, β_1 is the linear projection parameter vector. In addition, equation (7) is continuous with respect to β_1. Hence, by substituting this consistent estimate it is possible to make a consistent estimation of $E(WTP)$ based on equation (9). The estimation of the mean WTP based on equation (10) is even simpler. It is possible to make a consistent estimation as a sample average of y_{1i} weighted by the density function without regression.[5]

Furthermore, if the first bid follows uniform continuous distribution in the range of $[0, B]$, then equations (9) and (10) are as follows:

$$\hat{E}(WTP) = \hat{\beta}_{11} + \frac{B}{2}\hat{\beta}_{12} \cdots (11)$$

$$\hat{E}(WTP) = B\bar{y} \cdots (12)$$

Here, $\bar{y} \equiv n^{-1}\sum_{i=1}^{n} y_{1i}$ is the percentage of respondents who say "yes" for all bids. In particular, equation (12) makes it possible to estimate the mean WTP as "maximum bid" × "the percentage of respondents who say 'yes'." Thus, calculation becomes so simple that an estimate can be obtained even by manual calculation.

3.3 Problems of extending to the double-bounded dichotomous choice CV

When a linear projection model is applied to a double-bounded dichotomous choice CV, a bid provided for a respondent at the second stage depends upon an answer at the first stage. For this reason, the density function of the bid at the second stage cannot be known in advance. For this reason, it is not possible to apply a single-bound linear projection model to the double-bounded case without any modification. An Intuitive solution is used to estimate the density of a bid at the second stage from data. As shown in the Appendix, the use of kernel density estimation makes it possible to apply a double-bounded dichotomous choice linear projection model. The methodological simplicity, however, is lost due to the need for density estimation.

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[4] $y_{1i}/f_1(t_{1i})$ is independent, identically distributed. Hence, the weak law of large numbers implies that it is a consistent estimate of the mean WTP.

The second problem to be considered in extending to a double-bounded CV is that an answer at the second stage is not independent of an answer of the first stage. It is therefore not possible to apply the law of large numbers in the same way as in Section 3.2. Thus, consistency of the mean WTP cannot be guaranteed.

In order to deal with the two problems described above, a method that consistently estimates the mean WTP is developed in the next section.

4. Extension to double-bounded CV

4.1 Density of the second bid

The first, second lower, and second higher bid are notated as t_1, t_{2L}, and t_{2U}, respectively. If a respondent says "yes" at the first stage, t_{2U} is provided at the second stage. If a respondent says "no" at the first stage, t_{2L} is provided at the second stage. Thus, an actual bid provided for a respondent is either t_{2L} or t_{2U}, meaning that the answer of the second bid is dependent on the answer of the first bid. For this reason, even if the design is constructed so that the second bids, t_{2L} and t_{2U}, follow uniform distribution, the second bid provided for a respondent does not actually follow uniform distribution, making it impossible to control distribution of the second bid. It is the issue that is pointed out in the previous section.

Changing the view, consider that with no regard to an answer of "yes" or "no" at the first stage, both of the bid, $t_{2L}$and t_{2U}, are virtually provided at the second stage. This explains that t_{2U} is actually provided at the second stage for a respondent who says "yes" at the first stage, but at the same time, t_{2L} is also virtually provided at the second stage. Though a respondent does not directly answer to t_{2L}, it is possible to assume logically that a respondent will say "yes" for t_{2L} as well, because t_{2L} is lower than t_1 due to the design of the bid. On the other hand, t_{2L} is actually provided for a respondent who says "no" at the first stage. Similarly, considering that t_{2U} is also provided virtually, it allows us to assume logically that the answer to t_{2U} will be "no". This idea makes it possible to totally control the probability distribution of t_{2L} and t_{2U} at the second stage and extend a linear projection model to double-bounded case without density estimation. Now that it is possible to totally control the second bid by following the logic mentioned above, a linear projection model for double-bounded dichotomous choice will be developed.

4.2 Linear projection model for double-bounded dichotomous choice

An intuitive idea for the extension of a linear projection model to the double-bounded case is to take linear projection separately at the first and second stage. The indicator function y_1, y_{2L}, y_{2U} is defined as the following; if a respondent says “yes” for t_1, t_{2L}, t_{2U}, it is 1; if a respondent says “no,” it is 0. Also, the density of t_1, t_{2L}, t_{2U} is defined as f_1, f_{2L}, f_{2U}. Next, we consider the following linear projection.

$$LP(z|X) = X\beta \Leftrightarrow z = X\beta + v, E\left(X'v\right) = 0 \cdots (13)$$

$$z \equiv \begin{pmatrix} \frac{y_1}{f_1} \\ \frac{y_{2L}}{f_{2L}} \\ \frac{y_{2U}}{f_{2U}} \end{pmatrix}, X \equiv \begin{pmatrix} x_1' & 0 & 0 \\ 0 & x_{2L}' & 0 \\ 0 & 0 & x_{2U}' \end{pmatrix}, x_k \equiv \begin{pmatrix} 1 \\ t_k \end{pmatrix}, k = 1, 2L, 2U$$

where $\beta \equiv \begin{pmatrix} \beta_1 \\ \beta_{2L} \\ \beta_{2U} \end{pmatrix}, \beta_k \equiv \begin{pmatrix} \beta_{k1} \\ \beta_{k2} \end{pmatrix}$, and $v \equiv \begin{pmatrix} v_1 \\ v_{2L} \\ v_{2U} \end{pmatrix}$ are projection parameter vector and projection error, respectively. As is the case with equations (7) and (8), it is possible to calculate the mean WTP using the projection vector of the first stage and that of the second stage.

$$E(WTP) = \int_0^B (\beta_{k1} + \beta_{k2}t) f_k(t)dt + \int_0^B E(v_k|t) f_k(t)dt = \int_0^B (\beta_{k1} + \beta_{k2}t) f_k(t)dt \cdots (14)$$

$$E(WTP) = E\left(x_k'\beta_k\right) = E\left(\frac{y_k}{f_k}\right) \cdots (15)$$

To consider estimation, we add i to the subscript and rewrite equation (13): $z_i = X_i\beta + v_i, E\left(X_i'v_i\right) = 0$. Since $E(X_i'X_i)$ is a full column rank, $\beta = E\left(X_i'X_i\right)^{-1} E\left(X_i'z_i\right)$ is obtained. According to the analog principle, $\hat{\beta} = \left(n^{-1}\sum_{i=1}^{n} X_i'X_i\right)^{-1}\left(n^{-1}\sum_{i=1}^{n} X_i'z_i\right)$ is considered as this natural estimator.[6] Note that each element is an ordinary least squares estimator (OLSE), which regress $\frac{y_k}{f_k}$ on x_k at each stage, based on the structure of X_i and z_i. Thus, it is possible to make an estimation by performing simple regressions separately at each stage. It should be noted here that because i is randomly drawn, i is independent, identically distributed, even though the first and second stage are not independent.[7] Hence, $plim\,\hat{\beta} = \beta$ is obtained.[8] Consequently, the following consistent estimators of the mean WTP are derived by substituting these consistent estimates to equation (14) or by taking a sample average for equation (15):

$$\hat{E}(WTP) = \frac{1}{3}\sum_k \int_0^B \left(\hat{\beta}_{k1} + \hat{\beta}_{k2}t\right) f_k(t)dt \cdots (16)$$

$$\hat{E}(WTP) = \frac{1}{3}\left(\frac{1}{n}\sum_{i=1}^{n}\frac{y_{1i}}{f_1(t_{1i})} + \frac{1}{n}\sum_{i=1}^{n}\frac{y_{2Li}}{f_{2L}(t_{2Li})} + \frac{1}{n}\sum_{i=1}^{n}\frac{y_{2Ui}}{f_{2U}(t_{2Ui})}\right) = \frac{1}{3n}\sum_k\sum_{i=1}^{n}\frac{y_{ki}}{f_k(t_{ki})} \cdots (17)$$

It is possible to estimate the mean WTP estimator of equation (17) as the sample average of y_{ki} weighted by the density function without regression.

In the explanation above, the linear projection was taken separately at each stage. Yet, it is also possible to describe $X \equiv \begin{pmatrix} x_1' \\ x_{2L}' \\ x_{2U}' \end{pmatrix}$, $\gamma \equiv \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix}$, $u \equiv \begin{pmatrix} u_1 \\ u_{2L} \\ u_{2U} \end{pmatrix}$, and change the way to take the following projection:

$$z = X\gamma + u, E(X'u) = 0 \cdots (18)$$

Following the same procedure, consider $\hat{\gamma} = \left(n^{-1}\sum_{i=1}^{n} X_i'X_i\right)^{-1}\left(n^{-1}\sum_{i=1}^{n} X_i'z_i\right)$ as a natural estimator of $\gamma = E\left(X_i'X_i\right)^{-1}E(X_i'z_i)$. At this point, $plim\,\hat{\gamma} = \gamma$ is derived from the weak law of

[5] This looks like a seemingly unrelated regression (SUR). In the case of SUR, however, homoskedasticity is generally assumed for the error term (e.g., Hayashi, 2000, pp. 274–286). No assumption is applied to projection error here other than equation (13). Thus, this is not notated as SUR.

[6] The non-parametric model assumes that answers at the first and second stage are independent. In reality, however, these are obviously dependent.

[7] $\hat{\beta} = \beta + \left(n^{-1}\sum_{i=1}^{n} X_i'X_i\right)^{-1}\left(n^{-1}\sum_{i=1}^{n} X_i'v_i\right)$. Since i is independent, identically distributed, $plim\,\hat{\beta} = \beta$ from the weak law of large numbers and the definition of linear projection.

large numbers and the definition of linear projection. $\hat{\gamma}$ is the pooled ordinary least squares estimator (POLSE), which can be obtained through the simple regression of y_k/f_k and x_k pooled at each stage. Using this estimates, $\hat{\gamma}$, the following the mean WTP estimator is obtained.

$$\hat{E}(WTP) = \frac{1}{3}\sum_k \int_0^B (\hat{\gamma}_1 + \hat{\gamma}_2 t)\, f_k(t)dt \cdots (19)$$

In addition, even if the linear projection described in equation (18) is taken, there is no change in the mean WTP estimator based on weighted average corresponding to equation (17). In addition, if a bid follows uniform distribution, the mean WTP estimator corresponding to equations (16), (17), and (19) can be represented by extremely simple equations, such as (16)', (17)', and (19)' as shown below.

$$\hat{E}(WTP) = \frac{1}{3}\sum_k \left(\hat{\beta}_{k1} + \frac{B}{2}\hat{\beta}_{k2}\right) \cdots (16)'$$

$$\hat{E}(WTP) = B\bar{y}\,,\ \ \bar{y} \equiv \left(\frac{1}{3n}\sum_k \sum_{i=1}^{n} y_{ki}\right) \cdots (17)'$$

$$\hat{E}(WTP) = \hat{\gamma}_1 + \frac{B}{2}\hat{\gamma}_2 \cdots (19)'$$

Equation (17)', $\bar{y} \equiv \left(\frac{1}{3n}\sum_k \sum_{i=1}^{n} y_{ki}\right)$ is the percentage of respondents who say "yes" in all stages. Hence, the mean WTP is consistently estimated by "maximum bid" × "percentage of respondents who say yes." Thus, it is easier to estimate the mean WTP than any other parametric or non-parametric models.

Table 1 and Table 2 show the summary of the mean WTP estimator of a linear projection model of the double-bounded dichotomous choice CV.

Table 1 Mean WTP estimator of a double-bounded linear projection model

sample average type: equation (17)	regression type: equation (16)	regression POLS type: equation (19)
$\frac{1}{3n}\sum_k \sum_{i=1}^{n} \frac{y_{ki}}{f_k(t_{ki})}$	$\frac{1}{3}\sum_k \int_0^B \left(\hat{\beta}_{k1} + \hat{\beta}_{k2} t\right) f_k(t)dt$	$\frac{1}{3}\sum_k \int_0^B (\hat{\gamma}_1 + \hat{\gamma}_2 t) f_k(t)dt$

Note 1) $k = 1, 2U, 2L$ defines the first, second lower, and second higher bid, respectively.

Note 2) $\hat{\beta}_{k1}$ and $\hat{\beta}_{k2}$ are OLSE obtained through simple regression performed separately at each stage. $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are POLSE obtained through pooled simple regression.

Table 2 Mean WTP estimator of a double-bounded linear projection model

(if a bid at each stage follows uniform distribution)

sample average type: equation (17)’	regression type: equation (16)’	regression POLS type: equation (19)’
$B\bar{y}$	$\frac{1}{3}\sum_k \left(\hat{\beta}_{k1} + \frac{B}{2}\hat{\beta}_{k2}\right)$	$\hat{\gamma}_1 + \frac{B}{2}\hat{\gamma}_2$

Note 1) $k = 1, 2U, 2L$ defines the first, second lower, and second higher bid.

Note 2) $\hat{\beta}_{k1}$ and $\hat{\beta}_{k2}$ are OLSE obtained through simple regression performed separately at each stage. $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are POLSE obtained through pooled simple regression.

Note 3) $\bar{y} \equiv \left(\frac{1}{3n}\sum_k \sum_{i=1}^{n} y_{ki}\right)$ is the percentage of respondents who say “yes” in all stages.

5. Monte Carlo simulations

Theoretically, a bid is distributed within the support of population WTP and the density must not be 0 in the area. In actual application, however, a bid is designed to follow discrete uniform distribution. Hence, performance of the mean WTP estimator is examined when a bid is in a discrete uniform distribution. We also compare the linear projection model with a parametric model and a nonparametric model: the kernel regression model.[9]

It is assumed that the population WTP follows lognormal distribution, $\ln(WTP) \sim N(3,1)$. Here, the true mean WTP is 33.12 and the standard deviation is 43.41. Pairs of bids, t_1, t_{2L}, and t_{2U} are designed to follow discrete uniform distribution at each stage, as shown in Table 3 and Table 4. If the number of bid levels at the first stage is 5, one of the pairs between patterns 1 and 5 is provided with equal probability (Table 3). On the other hand, if it is 10, one of the pairs between patterns 1 and 10 is provided with equal probability (Table 4). In addition, the number of samples is 300 and the number of Monte Carlo simulation trials is 3000.

Upon estimation, it should be noted that the density of a bid, $f_k(t_k)$, must be $\frac{1}{B}$ and not $\frac{1}{\text{Number of bid levels}}$. This is because equation (14) cannot be formed unless $f_k(t_k)$ is $\frac{1}{B}$.[10]

The results of the Monte Carlo simulations are described in Table 5 and Table 6. If the number of bid levels at the first stage is 10, the mean WTP is estimated accurately enough, as shown in Table 6. Moreover, the linear projection model is slightly more efficient than both the parametric model, which adopts the maximum-likelihood method, and the non-parametric model. On the other hand, if the number of bid levels at the first stage is 5, the performance of a sample average type estimator (equation (17)') is a little bit poor. It is, therefore, necessary to be careful to set the number of bid levels when using a linear projection model.[11] For example, as is the case with the Monte Carlo simulation settings, if the WTP ranges from 0 yen to 200 euro, it is required to set the number of bid levels at approximately 10. In other words, it is necessary to distribute the bid comprehensively in the range of 0 yen to 200 euro, in increments of approximately 20 euro. Such a number of bid levels, however, is not very large, and it would therefore be fair to say that the

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[9] In the kernel regression model, the kernel function is Gaussian and the bandwidth is a plug-in estimate.

[10] Considering that a bid is distributed (virtually) based on continuous uniform distribution in population and a part of the bid is drawn randomly, this point is natural. This idea would enable us to understand that a bid must be distributed as comprehensively as possible in order to reproduce the characteristics of population in a sample.

[11] The influence of designing the number of bid levels and the maximum bid on mean WTP estimates is analyzed in Watanabe and Asano (forthcoming) at length.

linear projection model is a realistic approach.

6. Conclusion

This paper has extended the linear projection model of the dichotomous choice CV developed by Watanabe and Asano (forthcoming) to a double-bounded model. It has shown that as long as the bid follows uniform distribution, it is possible to make a consistent estimation of the mean WTP just by calculating the “maximum bid” × “percentage of respondents who say “yes”.” This model eliminates the risk in specification error. It also makes it easier than any other existing parametric or non-parametric models do to make a consistent estimation of the mean WTP. In order to ensure the transparency of the evaluation and improve its reliability, it is better to estimate the mean WTP based on as simple a method as possible. With the method developed in this paper, there is little room for arbitrariness in estimation at a practical stage. A bid, however, must be prudently designed. It is necessary to provide a bid comprehensively in the support of WTP distribution based on as many levels as possible. Nevertheless, based on the Monte Carlo simulation, it has been proven that it is possible to estimate the mean WTP accurately enough, even if the number of bid levels is not very large. It would be fair to say that the application of a linear projection model is realistic.

Finally, we will briefly explain the limitations of a linear projection model. First, only the mean WTP can be estimated with a linear projection model; it is not possible to estimate other representative values such as median. Yet, considering that one of the main purposes of contingent valuation is to apply it to cost-benefit analysis based on the potential compensation principle, it would be fair to say that the estimation of the mean WTP is sufficient in such a case. In addition, the model suggested in this paper cannot include any socioeconomic variables that influence the WTP. We expect that this extension will be achieved in the future.

Appendix: Linear projection model that adopts density estimation

Here, we show that the mean WTP is consistently estimated by the linear projection model by applying kernel estimation to make a consistent estimation of the density of the second bid a respondent faces actually. In kernel estimation, the consistency condition is not strong. The condition to make a consistent estimation by the kernel density estimation is as follows.

As the number of samples increases, the bandwidth becomes narrower, although the degree of decrease of the bandwidth must be smaller than the degree of increase of the number of samples: if $n \to \infty$, $h(n) \to 0$ and $nh(n) \to \infty$, where n is the number of samples and $h(n)$ is bandwidth.[12] Notating z_i as $\hat{z}_i$, when an estimate obtained through kernel density estimation is used, I will show that if $\hat{z}_i \xrightarrow{p} z_i \ (i = 1,2,\cdots n)$, $\hat{\beta} = \left(n^{-1}\sum_{i=1}^{n} x_i\, x_i^{'}\right)^{-1}\left(n^{-1}\sum_{i=1}^{n} x_i \hat{z}_i\right) \xrightarrow{p} E\left(x_i x_i^{'}\right)^{-1} E(x_i z_i) = \beta$.[13]

$\hat{z}_i \xrightarrow{p} z_i$ is given. Hence, by Slutsky's theorem,

$$\forall\epsilon\exists N_1 : n \geq N_1 \Rightarrow Pr\left(\left|n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - n^{-1}\sum_{i=1}^{n} x_i z_i\right| > \frac{\epsilon}{2}\right) = 0 \cdots (20)$$

In addition, assuming that any appropriate moment exists, by the weak law of large numbers,

$$\forall\epsilon\exists N_2 : n \geq N_2 \Rightarrow Pr\left(\left|n^{-1}\sum_{i=1}^{n} x_i z_i - E(x_i z_i)\right| > \frac{\epsilon}{2}\right) = 0 \cdots (21)$$

Here, $|n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - E(x_i z_i)| \leq |n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - n^{-1}\sum_{i=1}^{n} x_i z_i| + |n^{-1}\sum_{i=1}^{n} x_i z_i - E(x_i z_i)|$ is given. Hence, if $|n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - n^{-1}\sum_{i=1}^{n} x_i z_i| \leq \frac{\epsilon}{2}$ and $|n^{-1}\sum_{i=1}^{n} x_i z_i - E(x_i z_i)| \leq \frac{\epsilon}{2}$, $|n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - E(x_i z_i)| \leq \epsilon$.

The contrapositive of this is $|n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - E(x_i z_i)| > \epsilon \Rightarrow$ $|n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - n^{-1}\sum_{i=1}^{n} x_i z_i| > \frac{\epsilon}{2}$ or $|n^{-1}\sum_{i=1}^{n} x_i z_i - E(x_i z_i)| > \frac{\epsilon}{2}$. Therefore, $\{|n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - E(x_i z_i)| > \epsilon\} \subset \left\{|n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - n^{-1}\sum_{i=1}^{n} x_i z_i| > \frac{\epsilon}{2}\right\} \cup \left\{|n^{-1}\sum_{i=1}^{n} x_i z_i - E(x_i z_i)| > \frac{\epsilon}{2}\right\}$. Hence,

[12] See Silverman (1986) pp. 70–74.

[13] Here, to avoid notational complications, I have focused on the regression case at each stage as shown in section 4.2. This, however, applies to other cases as well. In addition, an estimator of the sample mean type is trivial, so we have not proven it here.

$Pr\{|n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - E(x_i z_i)| > \epsilon\} \leq \Pr\left\{|n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - n^{-1}\sum_{i=1}^{n} x_i z_i| > \frac{\epsilon}{2}\right\} + Pr\left\{|n^{-1}\sum_{i=1}^{n} x_i z_i - E(x_i z_i)| > \frac{\epsilon}{2}\right\}$. Here, let $N \equiv max(N_1, N_2)$, by equations (20) and (21), $n \geq N \Rightarrow Pr\{|n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i - E(x_i z_i)| > \epsilon\} = 0$. Therefore, if $\hat{z}_i \xrightarrow{p} z_i$, $(n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i) \xrightarrow{p} E(x_i z_i)$. Hence, if $\hat{z}_i \xrightarrow{p} z_i$, $(n^{-1}\sum_{i=1}^{n} x_i x'_i)^{-1} (n^{-1}\sum_{i=1}^{n} x_i\hat{z}_i) \xrightarrow{p} E(x_i x'_i)^{-1}E(x_i z_i)$,because $(n^{-1}\sum_{i=1}^{n} x_i x'_i)^{-1} \xrightarrow{p} E(x_i x'_i)^{-1}$. For this reason, even if the consistent estimate of the density of a bid at the second stage is used, this does not impair the consistency of $\hat{\beta}$. Hence, it is possible to make a consistent estimation of the mean WTP in a linear projection model using kernel estimation as well.

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Table 3 Bid design -the number of bid levels at the first stage is 5-

pattern	t1	t2L	t2U
1	5	0	20
2	50	45	65
3	95	90	110
4	140	135	155
5	185	180	200

Table 4 Bid design -the number of bid levels at the first stage is 10-

pattern	t1	t2L	t2U
1	5	0	20
2	25	20	40
3	45	40	60
4	65	60	80
5	85	80	100
6	105	100	120
7	125	120	140
8	145	140	160
9	165	160	180
10	185	180	200

Table 5 Result of the Monte Carlo simulation

-the number of bid levels at the first stage is 5-

	standard deviation	10% tile	50% tile	90% tile
linear projection model				
sample average type equation (17)	2.48	24.92	28.01	31.26
regression type equation(16)	3.12	29.18	33.00	37.16
regression POLS type equation(19)	2.78	30.35	33.82	37.38
parametric model	2.84	30.42	34.04	37.76
nonparametric model (kenrnel regression)	2.96	28.03	31.72	35.59

Note) Population mean WTP is 33.12.

Table 6 Result of the Monte Carlo simulation

-the number of bid levels at the first stage is 10-

	standard deviation	10% tile	50% tile	90% tile
linear projection model				
sample average type equation (17)	2.75	28.27	31.65	35.34
regression type equation(16)	2.89	30.01	33.61	37.51
regression POLS type equation(19)	2.70	27.26	30.62	34.26
parametric model	2.91	28.34	31.96	35.78
nonparametric model (kenrnel regression)	2.98	27.55	31.22	35.22

Note) Population mean WTP is 33.12.