

# Neurofuzzy Portfolio Selection Policies in A Small National Assets Model of Japan\*

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## Abstract

This paper proposes the portfolio selection in National assets of Japan subject to national balance model using neurofuzzy approach. National assets includes the wide range assets from natural resources to financial assets so that asset selection has to be differentiated by their properties and attributes. Having divided by the individual group, the most influential variable has to be found by selecting the weights on hidden layer by the neural network between inputs and outputs. Three expected risks between the actual values of grouping assets, and the mean, desired and estimated values will be investigated and compared by minimizing their risks subject to the national balance model including some asset variables. The constraint of the balance model is constructed and estimated from data for the national flows of funds table. It is stated there exists the difference and characteristics between national asset selection and the corporation by using an illustrative application to national balance model of Japan.

Keywords: Neurofuzzy, portfolio selection, expected risk, national balance models.

## 1. Historical Background

Recently, there has been developed for theory of securities investment and its practical business application. In Japan, it has been tried to introduce into investment decision in Japan that investment techniques who organized investors looked for more efficient method of fund operation in investment society such as United States so that the financial assets has been greatly increased and deregulation has made a progress since the latter half of 1980's. There are two investment methods as: (i) the evaluation method for individual security based on fundamental analysis proposed by Graham-Dodd in 1930's (Graham and Dodd, 1934), (ii) modern portfolio theory developed by (Markowitz, 1952) and (Sharpe, 1956) since 1950's. This paper deals with the Portfolio Selection theory based on the latter idea proposed paper by Markowitz. The theory is explained by the market equilibrium that the securities investor faces asset prices under the risk.

## 2. Modeling by Mean-Variance Analysis

Mean-Variance method is modeled by maximizing the expected values of rate of return for assets and minimizing the variance (risk) obtained from the time-series data for the various

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factors.

It is assumed that the time-series data of yearly rate of return for any  $i$  asset for all  $n$  assets in the period  $T$  is as follows.

$$a_{i1}, a_{i2}, \dots, a_{iT} \quad (i=1, 2, \dots, n) \quad (1)$$

The expected asset value  $\mu_i$  is expressed by

$$\mu_i = \frac{1}{T}(a_{i1} + a_{i2} + \dots + a_{iT}) \quad (2)$$

Let  $x_i$  as investment of  $i$  asset. We can obtain the time-series return of asset as follows:

$$\sum_{i=1}^n x_i a_{i1}, \sum_{i=1}^n x_i a_{i2}, \dots, \sum_{i=1}^n x_i a_{iT} \quad (3)$$

Assuming that the investor ratio invested diversely in the past, we can write the expected value of the aggregated investment as:

$$E(x) = \sum_{i=1}^n \mu_i x_i \quad (4)$$

Further the variance (risk)  $V(x)$  is expressed by

$$V(x) = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j \quad (5)$$

where the  $\sigma_{ij}$  is

$$\sigma_{ij} = \frac{1}{T} \sum_{t=1}^T (a_{it} - \mu_i)(a_{jt} - \mu_j) \quad (6)$$

Therefore, Mean-Variance method is formulated as follows:

$$\text{Maximize } E(x) = \sum_{i=1}^n \mu_i x_i \quad (7)$$

$$\text{Minimize } V(x) = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j \quad (8)$$

$$\text{subject to } \sum_{i=1}^n x_i = 1, \text{ where } x_i \geq 0 \quad (i=1, 2, \dots, n) \quad (9)$$

In order to avoid optimization of two objectives, we reformulate minimization of investment risk subject to a constant expected return of asset according to Markowitz as follows:

$$\text{Minimize } V(x) = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j \quad (10)$$

$$\text{subject to } \sum_{i=1}^n \mu_i x_i = E \quad (11)$$

$$\sum_{i=1}^n x_i = 1, \text{ where } x_i \geq 0 \quad (i=1, 2, \dots, n) \quad (12)$$

This implies the risk minimization by diversifying investments of assets. According to the traditional portfolio theory assumed to be the investor behaves as an risk-avertter, the desirable or optimal solution is obtained by taking the most efficient combination for the obtainable M-V combinations area with the convex utility function to below.

### 3. Neurofuzzy Portfolio Selection Algorithm

Let us consider the following estimated nonlinear econometric models in terms of ARMAX vector-type including the linguistic variable values:

$$\mathbf{y}_t = f\{\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \dots, \mathbf{x}_{t-1}, \mathbf{x}_{t-2}, \dots, \mathbf{z}_t, \mathbf{z}_{t-1}, \mathbf{z}_{t-2}, \dots, \mathbf{e}_t\} \quad (13)$$

where  $\mathbf{y}_t$ : target vector,  $\mathbf{x}_t$ : control vector,  $\mathbf{z}_t$ : data vector, and  $\mathbf{e}_t$ : residual vector.

This section formulates defuzzification using linguistic variables. First, fuzzy logic inference ultimately gives the fuzzy rules as the relations of the linguistic variable values between the antecedents and consequents. The section is to deal with the basic framework and rewrite as

$$\mathbf{y}_t = \mathbf{f}(\mathbf{Y}_t) + \mathbf{e}_t \quad (14)$$

Assume the linguistic representation of model (1) is given by a set of fuzzy rules, the  $ij$ -th fuzzy rule is

$$r_{ij}: \text{IF } (\mathbf{Y}_t \text{ is } \mathbf{A}^i) \text{ THEN } (\mathbf{y}_t \text{ is } \mathbf{B}^j) \quad (15)$$

where  $\mathbf{A}^i$  is the multivariate fuzzy set obtained from the intersection of  $k$  univariate fuzzy sets of each input, and  $\mathbf{B}^j$  is the univariate fuzzy set of the output. The antecedent  $(\mathbf{Y}_t \text{ is } \mathbf{A}^i)$  related to every possible consequent of  $(\mathbf{y}_t \text{ is } \mathbf{B}^j)$  ( $j=1, 2, \dots$ ). This can be shown by the following examples:

Rule 1: IF <Currency and deposits=very low> THEN <Financial Assets=very low>

Rule 2: IF <Intangible Fixed Assets=very high> THEN <Total assets=very high>

Antecedent Part:

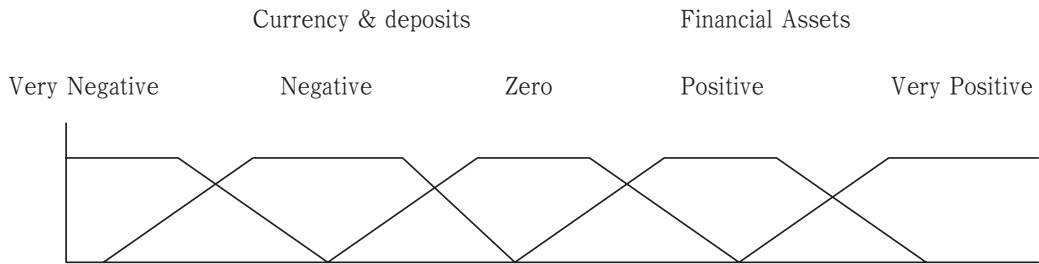


Fig. 1 Example of Simple Fuzzy Economic Rules and Its Corresponding Membership Function for Antecedent Clauses

Consequent Part:

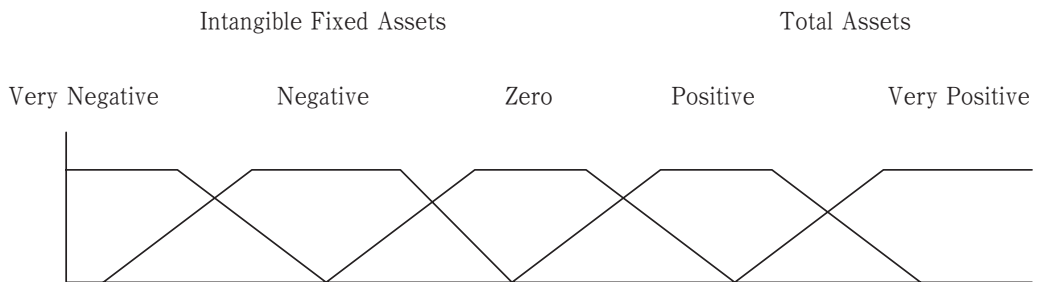


Fig. 2 Examples of Simple Fuzzy Economic Rules and Its Corresponding Membership Function for Consequent Clauses

The fuzzy output set is obtained from fuzzy rules using union operator. Assuming the real-valued inputs are represented by fuzzy singletons  $\mathbf{B}^j$ , being symmetric and bounded for all  $j$  with addition and product operators. Using the center of gravity defuzzification algorithm, the defuzzied output is

$$\nu_j = \prod_{i \in I} \nu_{ij} \quad (16)$$

$$y_t = \frac{\sum_{j=1}^J \nu_j w_{kj}}{\sum_{j=1}^J \nu_j} \quad (17)$$

where  $I$  is input number,  $J$  is fuzzy rule number  $\nu$  is the weights and  $k$  is the data number. The shape of membership function is the radial basis function with insensitive range  $c$  that is useful for reducing the membership functions. The membership function of an input value and the  $j$ -th fuzzy rule is expressed by

$$f(I_i) = \begin{cases} -b_{ij}(|I_i - a_{ij}|)^2 & \text{if } |I_i - a_{ij}| = c_{ij} \\ 0 & \text{if } |I_i - a_{ij}| = c_{ij} \end{cases} \quad (18)$$

$$m_{ij} = \exp\{f(I_i)\} \quad (19)$$

$$m_{ij} = \exp\{f(I_i)\} \quad (20)$$

The gradient decent method to tune antecedent and consequent parts of parameter variation rules are presented as follows:

Consequent Part:  $w_{ij} = w_{ij}^* - k_w \cdot \partial E_p / \partial w_{pj}$

where  $w$ ,  $p$ , and  $k_w$  are the consequent value, the rule number, and the learning coefficient, respectively.

Antecedent Part:

$$a_{ij} = a_{ij}^* - k_a \cdot \partial E_p / \partial a_{ij}, \quad b_{ij} = b_{ij}^* - k_b \cdot \partial E_p / \partial b_{ij}, \quad c_{ij} = c_{ij}^* - k_c \cdot \partial E_p / \partial c_{ij} \quad (21)$$

where  $k_a$ ,  $k_b$ ,  $k_c$  are the learning coefficients, and  $E_p$  is the deviation between actual target value and the desired, where  $a$ ,  $b$ ,  $c$  are the coefficients forming the shape of membership functions.

In the national assets and liabilities, first, we divide the assets into five kinds of assets such as intangible assets, tangible assets financial assets, net assets and liabilities because they have different properties and behaviors in the market economy. Second, MV portfolio analysis in general can be formulated as the minimization of the quadratic performance function represented by trade-off between return and risk subject to the national financial balance model consisting of asset, liability and stock variables. Therefore, we have to differentiate the performance functions for respective variable and target as follows: for tangible assets: minimi-

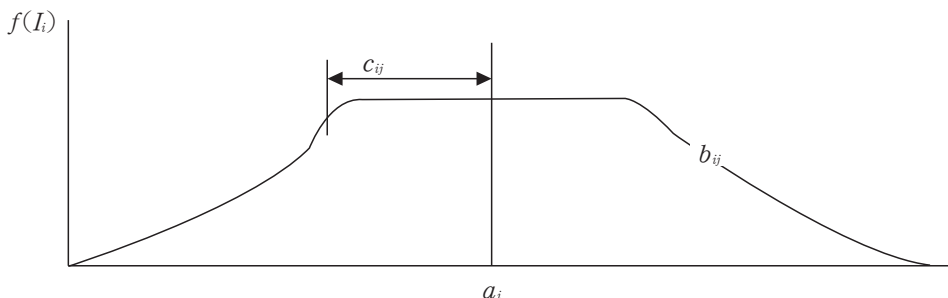


Fig. 3 Membership function by Radial Basis Function

zation of difference between desired assets and actual physical assets, for financial assets: minimization of difference between actual assets and mean, which is indeed MV-analysis, and for liabilities: minimization of difference between estimated liabilities and the actual.

The general structure and functions represented by neurofuzzy control network, which is a feedforward multilayered connectionist structure to realize the traditional fuzzy logic control systems from sets of input-output training data. The typical network is represented by weight values between one layer unit and the other. The integrated function for between in- and out-signals through nodes can be represented by

$$\text{net-input} = f(x_1^k, x_2^k, \dots, x_p^k; w_1^k, w_2^k, \dots, w_p^k) \quad (22)$$

where  $x_i^k$  is the  $i$ -th signal at the  $k$ -th layer,  $w_i^k$  the  $i$ -th link weight of the  $k$ -th layer,  $k$  indicates the layer number,  $p$  the number of a node's input connections, and  $f$  the function combining information, activation or evidence between signals. The outputs are activated by the net inputs as follows:

$$\text{output} = o_i^k = a(f) \quad (23)$$

where  $a(f)$  presents the activation function representing the typical sigmoid function as:

$$f = \sum_{i=1}^p w_i^k x_i^k \text{ and } a = \frac{1}{1 + e^{-f}} \quad (24)$$

for the bell-shaped function:

$$f = \sum_{i=1}^p w_i^k x_i^k = \sum_{i=1}^p \mu_{ij} \sigma_{ij} x_i^k \text{ and } a = \frac{f}{\sum \sigma_{ij} x_i^k}, \quad (25)$$

where the link weight at layer  $k$  ( $w_{ij}^k$ ) is  $\mu_{ij} \sigma_{ij}$ . It implies that the learning algorithm includes structure and parameter learning to optimal means ( $\mu_{ij}$ 's) and standard deviation ( $\sigma_{ij}$ 's) of nodes in layers. That is, it will learn fuzzy logic rules by determining the antecedent and consequent links of the rule nodes.

Having established the fuzzy logic rules and the whole net work structure, the neural networks are utilized to adjust the parameters of the member ship function optimally. The problem is formulated that giving the training input data  $x_{it}$ ,  $i = 1, \dots, n$ , the desired output values  $y_{it}$ ,  $i = 1, \dots, m$ , the number of the fuzzy partitions of input state linguistic variable  $x_i$ ,  $n(x_i)$ , and the fuzzy logic rules, the parameters of the membership functions are optimally adjusted. The idea of back-propagation is to minimize the error function:

$$E = \frac{1}{2} (y_t - \hat{y}_t)^2, \quad (26)$$

where  $y_t$  is the actual output. The learning process is as follows: For each training data set, letting start at input, a forward pass is used to compute the activity levels of all nodes in the network, and then starting at the output nodes, a backward pass is used to compute  $\partial E / \partial y$  for all the hidden nodes. Assuming that the bell-shaped membership functions with mean's,  $\mu_i$ 's and standard deviation's,  $\sigma_i$ 's as the adjustable weighting parameters for these computations instead of generally using the weighting values  $w_i$  as above.

The adaptive rule of the mean  $m_i$  is derived as:

$$\frac{\partial E}{\partial \mu_i} = \frac{\partial E}{\partial a^k} \cdot \frac{\partial a^k}{\partial \mu_i} = - (y_t - \hat{y}_t) \cdot \frac{\sigma_i x_i^k}{\sum \sigma_i x_i^k} \quad (27)$$

Hence, the mean parameter is recursively computed by

$$\mu_{it+1} = \mu_{it} + \eta(y_t - \hat{y}_t) \cdot \frac{\sigma_i x_i^k}{\sum \sigma_i x_i^k} \quad (28)$$

Similarly, the same adaptive rule of the deviation  $s_i$  can be obtained as:

$$\frac{\partial E}{\partial \sigma_i} = \frac{\partial E}{\partial a^k} \cdot \frac{\partial a^k}{\partial \sigma_i} = -(y_t - \hat{y}_t) \cdot \frac{\mu_i x_i (\sum \sigma_i x_i^k) - (\sum \mu_i \sigma_i x_i^k) x_i^k}{(\sum \sigma_i x_i^k)^2} \quad (29)$$

Hence, the deviation parameter is recursively computed by

$$\sigma_{it+1} = \sigma_{it} + \lambda(y_t - \hat{y}_t) \cdot \frac{\mu_i x_i (\sum \sigma_i x_i^k) - (\sum \mu_i \sigma_i x_i^k) x_i^k}{(\sum \sigma_i x_i^k)^2} \quad (30)$$

The error to be propagated to the preceding layer is

$$\delta^k = -\frac{\partial E}{\partial a^k} = y_t - \hat{y}_t \quad (31)$$

Having computed the means and variances of each variable, we can adopt the adaptive rules to compute the weighting values.

#### 4. An illustrative Application

We show some financial equations from the national balance between assets and liabilities available from 'Closing Stocks, Capital Transactions and Reconciliations of Assets and Liabilities for the Nation' as the funds of financial flows in Japan. The equations have been estimated from the period data 1977 to 1998 taken from the Funds of Flows in Japan.

In this section it is shown the some asset equations taken from the national wealth model as examples to be applied for this method. These equations are selected from the national balance model proposed by the previous paper (Ito, 2003). It includes intangible asset variables, financial asset, liability and net worth. Some assets can be expressed by linguistic terms to be defuzzified to measure the quantitative values and the integrated. In this section the trajectories of the following four asset equation are computed by neurofuzzy method proposed. They are Cash and Deposits, Long-term Bond Securities, Financial Asset and Other Financial Asset equations. They have been regressed by least square method with goodness of fits and significant by t-test.

Stock equation

$$(1) \Delta \ln S = 0.0024 + 0.683986 \Delta \ln V + 0.204609 \Delta \ln S_{-1} + 0.175524 \Delta \ln R$$

Financial Asset equation

$$(2) \Delta \ln FA = -0.00928 + 0.750408 \Delta \ln AT + 0.987835 \Delta \ln CD - 0.99152 \Delta \ln (LT - CS) + 0.20613 \Delta \ln LI$$

Liabilities except for Corporate Shares:

$$(3) \Delta \ln (LT - CS) = -0.00342 + 0.022837 \Delta \ln OS - 1.2956 \Delta \ln CN + 2.311082 \Delta \ln TN,$$

where the above symbols are as follows:  $S$ =Stocks,  $R$ =interest rate,  $LT$ =total liabilities,  $CS$ =corporate shares,  $CN$ =Corporate Shares and Net Worth,  $V$ =Gross Domestic Products,  $AT$ =Total assets,  $TN$ =Total Liabilities and Net Worth,  $TS$ =total securities,  $FA$ =financial asset,  $LI$ =life insurance,  $OS$ =Operating Surplus, and  $NW$ =net worth. The target deviation is defined by  $d_t = y_t - \hat{y}_t$  is replaced by  $y_t - a$ , where  $a$  is a constant value so that  $d_t$  is also constant since  $y_t$  is logarithm form. Therefore, the minimization of error  $E$  is equal to zero.

Linguistic expressions by fuzzy logic corresponds these figures in the Table 1.

|             |               |          |      |          |               |
|-------------|---------------|----------|------|----------|---------------|
| Values (a)  | -0.04         | -0.02    | 0.0  | 0.02     | 0.04          |
| Percent     | -4 %          | -2 %     | 0 %  | 2 %      | 4 %           |
| Fuzzy Rules | Very Negative | Negative | Zero | Positive | Very Positive |

Table. 1 Linguistic expressions by Fuzzy Logic

The means  $\mu$ 's and variances  $\sigma$ 's of these variables are shown in the Table 2.

|             |                      |                 |                     |                      |                 |
|-------------|----------------------|-----------------|---------------------|----------------------|-----------------|
| (1)         | $\Delta \ln S$       | $\Delta \ln V$  | $\Delta \ln S_{-1}$ | $\Delta \ln R$       |                 |
| $\mu$ 's    | 0.02111              | 0.034291        | 0.02624             | -0.05762             |                 |
| $\sigma$ 's | 0.049465             | 0.022934        | 0.049052            | 0.144562             |                 |
| (2)         | $\Delta \ln FA$      | $\Delta \ln AT$ | $\Delta \ln CD$     | $\Delta \ln (LT-CS)$ | $\Delta \ln LI$ |
| $\mu$ 's    | 0.079184             | 0.071244        | 0.078482            | 0.078386             | 0.121117        |
| $\sigma$ 's | 0.05534              | 0.057743        | 0.034804            | 0.038103             | 0.048102        |
| (3)         | $\Delta \ln (LT-CS)$ | $\Delta \ln OS$ | $\Delta \ln CN$     | $\Delta \ln TN$      |                 |
| $\mu$ 's    | 0.049465             | 0.099026191     | 0.078385565         | 0.063770397          |                 |
| $\sigma$ 's | 0.066528403          | 0.072832913     | 0.03810288          | 0.067449148          |                 |

Table. 2 Means and Variances of Selected Assets Equations

$d_t = y_t - a$  :  $a$  = constant desired values of target variables in terms of growth rates are in the Table 1.

## 5. Preliminary for Experiment

Viewing the relations between original risks ( $\sigma$ ) and returns ( $\mu$ ) for national asset variables, we can observe that returns are relatively higher than risks, and returns and risks approximately range from  $-0.05$  to  $0.12$ , and from  $0.02$  to  $0.12$ , respectively in Fig. 4. The highest return is  $\Delta \ln (LT-CS)$  and the lowest with an negative sign is only  $\Delta \ln R$  while the highest risk is  $\Delta \ln R$  and the lowest  $\Delta \ln V$ . Returns and Risks for all asset variables are decreased as time goes. It seems that all growth rates of variables are converged as certain constants, however, there is not enough time for all variables to be converged completely for the certain period.

## 6. Experimental Design for Neurofuzzy Computation

In this section, it is shown that the experimental design for neuro-fuzzy computation of the various national wealth policies such optimal, MV and variance minimum policies. We can define the deviation  $d$  by fuzzy logic as follows:

$d_t = y_t - a$  :  $a$  = constant values of target variables in terms of growth rates as:

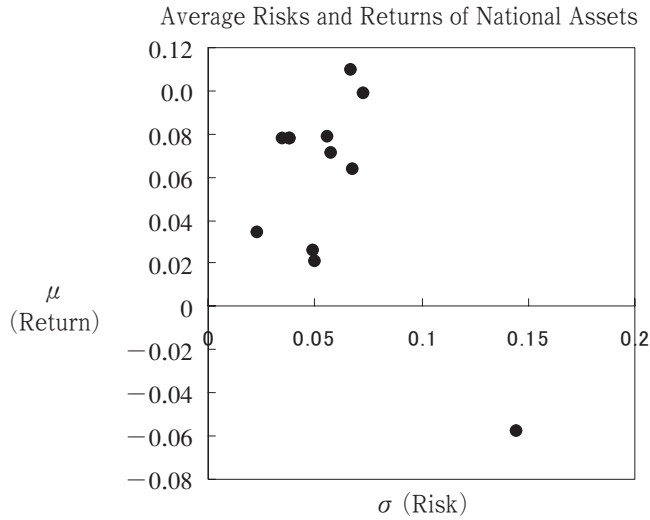


Fig. 4 Risks and Returns in Selected National Assets

|             |        |        |      |       |      |
|-------------|--------|--------|------|-------|------|
| a           | -0.04, | -0.02, | 0.0, | 0.02, | 0.04 |
| Percent     | -4 %,  | -2 %,  | 0 %, | 2 %   | 4 %  |
| Fuzzy Rules | VN     | N      | Z    | P     | VP   |

VN=Very Negative, N=Negative, Z=Zero, P= Positive, VP=Very Positive

The estimated equations are rewritten in terms of function types as,

Stock Policy:

$$(1) \Delta \ln S = f(\Delta \ln V, \Delta \ln S_{-1}, \Delta \ln R)$$

$$\Delta \Delta \ln V \rightarrow \Delta \ln S: \quad \partial y_i / \partial x_i = 0.683986$$

$$\Delta \Delta \ln S_{-1} \rightarrow \Delta \ln S: \quad \partial y_i / \partial x_i = 0.204609$$

$$\Delta \Delta \ln R \rightarrow \Delta \ln S: \quad \partial y_i / \partial x_i = 0.175524$$

$$f_k = \Delta \ln S$$

Financial Asset policy:

$$(2) \Delta \ln FA = f(\ln AT, \Delta \ln CD, \Delta \ln(LT-CS), \Delta \ln LI)$$

$$\Delta \Delta \ln AT \rightarrow \Delta \ln FA \quad \partial y_i / \partial x_i = 0.7504$$

$$\Delta \Delta \ln CD \rightarrow \Delta \ln FA \quad \partial y_i / \partial x_i = 0.9878$$

$$\Delta \Delta \ln(LT-CS) \rightarrow \Delta \ln FA \quad \partial y_i / \partial x_i = -0.9915$$

$$\Delta \Delta \ln LI \rightarrow \Delta \ln FA \quad \partial y_i / \partial x_i = 0.20613$$

$$f_k = \Delta \ln FA$$

Liability Policy:

$$(3) \Delta \ln(LT-CS) = f(\Delta \ln OS, \Delta \ln CN, \Delta \ln TN)$$

$$\Delta \Delta \ln OS \rightarrow \Delta \ln(LT-CS) \quad \partial y_i / \partial x_i = 0.022837$$

$$\Delta \Delta \ln CN \rightarrow \Delta \ln(LT-CS) \quad \partial y_i / \partial x_i = -1.2956$$

$$\Delta \Delta \ln TN \rightarrow \Delta \ln(LT-CS) \quad \partial y_i / \partial x_i = 2.311082$$

$$f_k = \Delta \ln(LT-CS)$$

Definition:

$$\begin{aligned}
 d_t &= \partial x_t / \partial f_k = \text{Actual} - \text{Desired} \rightarrow \text{Optimal Policy} \\
 &= \text{Actual} - \text{Mean} \rightarrow \text{MV Policy} \\
 &= \text{Actual} - \text{Estimated} \rightarrow \text{Residual Policy} \\
 \delta_k &= d_t \cdot \partial y_t / \partial x_t \cdot f_k (1 - f_k) \cdot \partial x_t / \partial f_k \\
 \mu_{k+1} &= \eta \delta_k f_k + \beta \mu_k
 \end{aligned}$$

where  $\eta$  = learning coefficient = 0.01 and  $\beta$  = momentum coefficient = 0.9  
for mean parameter  $\mu_h$

$$\begin{aligned}
 f_k &= \frac{\sigma_i x_i^k}{\sum \sigma_i x_i^k} \\
 \sigma_{k+1} &= \eta^* \delta_k^* f_k + \beta^* \sigma_k
 \end{aligned}$$

for deviation parameter  $\sigma_k$

$$f_k = \frac{\mu_i x_i (\sum \sigma_i x_i^k) - (\sum \mu_i \sigma_i x_i^k) x_i^k}{(\sum \sigma_i x_i^k)}$$

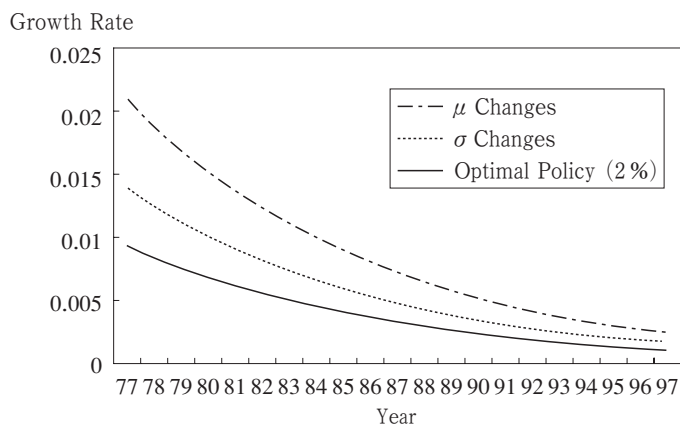
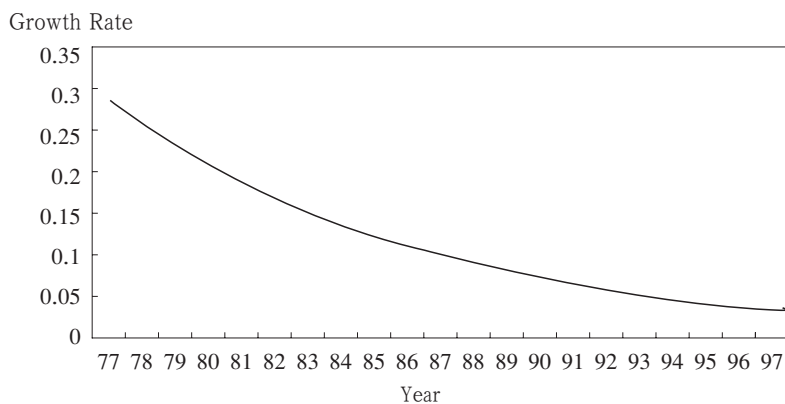
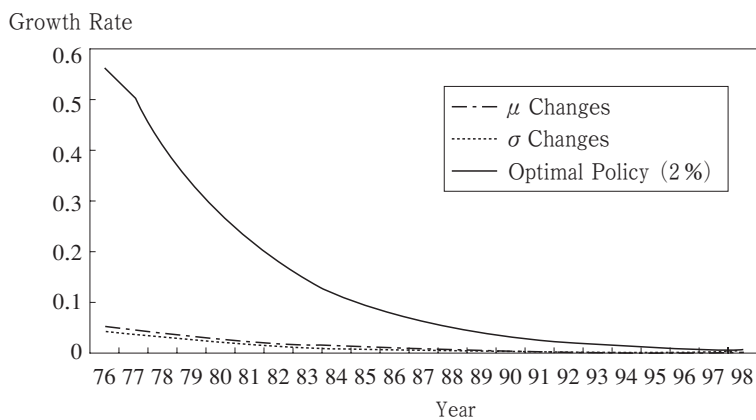
Stop condition

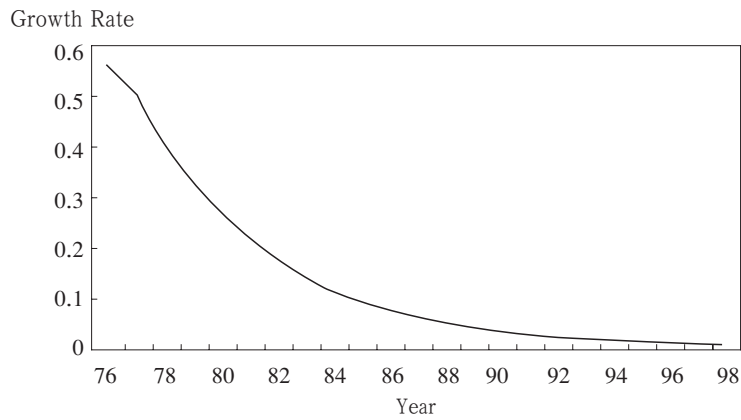
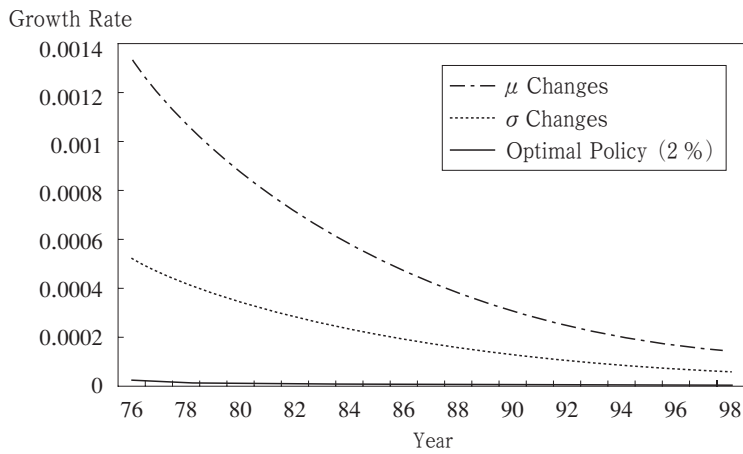
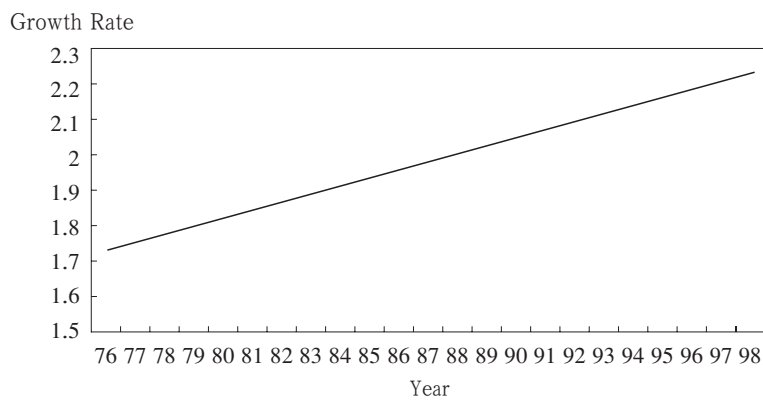
$$E = \frac{1}{2} d_i^2 \leq \varepsilon = 0.001$$

Five fuzzy rules are adopted, but it is only shown for 2% growth rate of GDP rule in the following. Fig. 5 shows optimal policy MV policy of  $\Delta \ln S$  is smaller than the MV policy, but all seems to converge to the certain values. In Fig. 6 Residual policy of  $\Delta \ln S$  shows the similar tendency, but the growth rate is bigger than former. The growth rates of the former decrease 2.2% to 0.25%. the later shows 35% decreasing to 4%. Fig. 7 shows optimal policy of  $\Delta \ln FA$  decreases from 57% to nearly zero, but the MV policy starts from the low growth level such as 5% to zero so that there is no big difference between the initial period to the final. For Fig. 8. Residual policy shows bigger difference between the initial period such as 55% to nearly zero toward the final period. Fig. 9 shows MV policy decreases bigger than optimal, but the values of growth rate for both are very small from 0.13% to zero. Particularly, the growth rate of  $\Delta \ln(LT-CS)$  by optimal policy is nearly to zero all the time. Fig. 10, Residual policy shows the growth rate of  $\Delta \ln(LT-CS)$  grows with constant rate. This is quite different from the cases of Short-term Borrowing and Financial assets.

## 7. Concluding Remarks

It is shown that in this section the national asset balance model of Japan is as an example for this method. This balance model has been used in the previous paper for Phillips-type stabilization policy (Ito, 2003), which includes mutually interacting intangible asset variables, the national financial asset, the net national asset and the liability. Some assets can be expressed by linguistic terms, to be defuzzified to measure the quantitative affect or integrated. The three-types of quadratic performance functions will be employed to compute the expected rates of return in comparison with Markowitz Model.

Fig. 5 MV and Optimal Policies for  $\Delta \ln S$ Fig. 6 Residual Policy for  $\Delta \ln S$ Fig. 7 MV and Optimal Policies for  $\Delta \ln FA$

Fig. 8 Residual Policy for  $\Delta \ln FA$ Fig. 9 MV and Optimal Policies for  $\Delta \ln(LT-CS)$ Fig. 10 Residual Policy for  $\Delta \ln(LT-CS)$

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