

The Effect of the Grant to the Local Government

— A Keynesian Case*

Yoshihiko Seoka

Summary

This paper presents a two region model with the local pure public goods in the Keynesian framework and examines how the grant from one region to another influences economic welfare in the whole economy. I obtain the following main results.

(i) The Keynesian model has the equilibrium which has inhabitants in both regions and is stable under given fiscal variables. (ii) In the decentralized economy suffering from shortage of effective demand in both regions, the grant from one region with the higher propensity to import to another with the lower propensity to import improves welfare. (iii) In the centralized economy full employment in the whole economy can be achieved with maximum welfare. But in this case there should be the grant from the region with lower propensity to import to the region with higher propensity to import.

Keywords: grants to the local government, increasing returns to scale,
stability of equilibrium, Keynesian economics, propensity to import.

[I] Introduction

This paper is based on objections to two conventional dichotomies in economics. The first is that the Keynesian macroeconomics deals with short run phenomena while the neoclassical economics investigates long run phenomena. In spite of the famous words of Keynes himself¹⁾ I contend that the problem of effective demand is also important in the long run. The second is that Keynesian economics is applied to the state of underemployment while neoclassical economics is applied to the state of full employment or at least to the state without shortage of effective demand. I contend that the state that there is no shortage of effective demand owing to some economic policy cannot be investigated in the

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1) "In the long run we are all dead", from *A Tract on the Monetary Reform* (1921) in *The Collected Writings of John Maynard Keynes*, vol IV. p.65,

neoclassical framework.

Nakajima (1995) induces me to write this paper. In the neoclassical framework he investigated the economy which consists of two regions and derived the various interesting results. Above all

(a) Under some condition there can exist the stable equilibrium with inhabitants in both regions. That is, the population will return to the equilibrium sooner or later if the population deviates at least a little from it.

(b) At the stable equilibrium the transfer from the overpopulated region to the underpopulated region increases welfare.

While the result (b) seems to be very attractive, however, the result (a) is crucially based on the assumption that there is congestion. This plays the role to eliminate one of the most important characteristics of public goods, non-competitiveness which is precisely one of the causes of instability²⁾.

In this paper I presents a Keynesian model which is based on the pure public goods in the sense that they have no congestion effect. In addition just as Davis & Weinstein (2002) emphasizes that regional allocation of economic activity depends on "locational fundamentals" as well as increasing returns, I also regard the interregional trade as an important element in the model. Thus I will derive the results similar to the Nakajima's paper but different in some important points.

Section 2 presents a Keynesian model with the stable equilibrium in which both regions have inhabitants under given fiscal variables. Section 3 presents a decentralized Keynesian model in which the local government can determine the rate of income tax and the amount of public expenditures while the central government can determine only the amount of transfer. Section 4 presents a centralized Keynesian model in which the central government can achieve full employment in the whole economy. Section 5 concludes and points out the problems which remain to be solved.

[II] A Keynesian Model with Two Regions

[2.1]

Two regions 1 and 2 exist in one economy. N_1 and N_2 denote the population (the number of households) in region 1 and 2 respectively and

$$(1) \quad N_1 + N_2 = N \text{ (constant).}$$

The utility of a household in region 1 is given by

$$(2) \quad U_1 = \alpha \ln c_1 + \beta \ln G_1,$$

2) In the real world, it is very difficult to find the public goods without congestion effect (one rare example is the radio-TV broadcasting by the government). But the pure public goods in this paper are rather imagined in order to focus on the core of the problem. Another well-known cause of concentration, increasing returns to scale of production in an original sense, is ignored in this paper, since taking it into account brings about nothing new.

where c_1 denotes the per capita consumption in region 1. G_1 denotes the amount of the local pure public goods in region 1 which are assumed to be effective in region 1 alone. Similarly for region 2, i.e.

$$(2)' \quad U_2 = \alpha \ln c_2 + \beta \ln G_2.$$

The household' budget constraint in each region is given by

$$(3) \quad c_1 N_1 = (1 - \tau_1) Y_1$$

and

$$(3)' \quad c_2 N_2 = (1 - \tau_2) Y_2,$$

where $\tau_i (i=1, 2)$ denotes the proportional income tax rate in the region i ³⁾ and Y_i , the total output in region i . And the local government's budget constraint in each region is given by

$$(4) \quad G_1 = \tau_1 Y_1 + T$$

and

$$(4)' \quad G_2 = \tau_2 Y_2 - T,$$

where T denotes transfer from region 2 to region 1. The production function in each region is given by

$$(5) \quad Y_1 = L_1$$

and

$$(5)' \quad Y_2 = L_2$$

where $L_i (i=1, 2)$ denotes labor input in region i .

Now if

$$(6) \quad U_1 = U_2$$

holds, a household in one region has no incentive to move to another region. In this sense this state can be called "equilibrium". However this equilibrium is unstable in the neoclassical case, as is well known.

In the Keynesian model, contrary to the neoclassical one, full employment does not always hold owing to shortage of effective demand (i.e. because of too small amount of government expenditures or too high rate of income tax). So from (3)~(5)' we obtain

$$(9) \quad N_1 \geq L_1 = Y_1 = t_1 (G_1 - T)$$

and

$$(9)' \quad N_2 \geq L_2 = Y_2 = t_2 (G_2 + T),$$

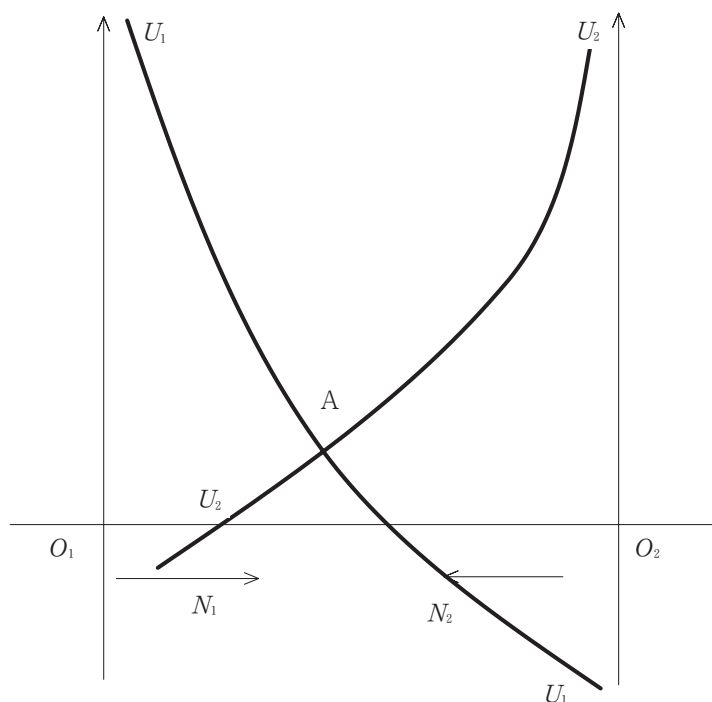
where $t_i = 1/\tau_i (i=1, 2)$ ⁴⁾. In this and next section, I will assume that strict inequality holds in (9) and (9)'. Then (2) and (2)' are reduced to the following equations.

$$(10) \quad U_1 = \alpha \ln (t_1 - 1) + \alpha \ln (G_1 - T) + \beta \ln G_1 - \alpha \ln N_1$$

3) The consumption tax (or added-value tax) can be easily introduced. But contrary to the traditional literature the lump-sum tax does not work.

4) (9) and (9)' show that an increase in T itself decreases output in the transferee (region 1) and increases it in the transferor (region 2). This is because, given τ_i and $G_i (i=1, 2)$, an increase in T means an increase in import (a fall in effective demand) in region 1 and an increase in export (a rise in effective demand) in region 2.

Fig. 1



$$(10)' \quad U_2 = \alpha \ln(t_2 - 1) + \alpha \ln(G_2 + T) + \beta \ln G_2 - \alpha \ln N_2$$

Fig. 1 illustrates (10) (curve U_1) and (10)' (curve U_2). In the figure, N_1 and N_2 are measured from O_1 and O_2 respectively. Two curves have the unique intersection point A (N_1^*, N_2^*). Notice that we assume that $N_1^* > L_1$ and that $N_2^* > L_2$. The point A is an equilibrium in the sense that (6), where U_1 and U_2 are given by (10) and (10)' respectively, holds and this equilibrium is stable in the following sense.

Suppose that the rate of change in N_1 is given by

$$(11) \quad dN_1(t)/dv = \delta (U_1(v) - U_2(v)),$$

where δ is a positive parameter and v denotes the *imaginary* time⁵⁾. Then, given the fiscal variables, if, for instance, $N_1 > N_1^*$, $U_1 < U_2$. Therefore N_1 will decrease and the economy will return to the equilibrium.

Thus we have the following proposition.

Proposition 1

Given all fiscal variables, in the Keynesian economy the equilibrium in which both regions

5) I admit that this method is very naive. See Tabuchi and Zeng(2001) who investigate the stability problem more sophiscatedly as well as in the general case with n regions.

have inhabitants is stable.⁶⁾ //

[2.2]

Now let us introduce the balance of interregional trade that

$$(12) \quad T = M_1 - M_2,$$

where M_i denotes imports of region i from another region. We assume that they are given by

$$(13) \quad M_1 = m_1 Y_1$$

and

$$(13)' \quad M_2 = m_2 Y_2,$$

where m_i ($0 < m_i < 1$, $i = 1, 2$) denotes “the propensity to import in region i ” in the usual sense.⁷⁾

Notice that even if (12), (13) and (13)' are introduced, Proposition 1 will still hold. Indeed from (9) and (9)' we obtain

$$(14) \quad T(1 + t_1 m_1 + t_2 m_2) = t_1 m_1 G_1 - t_2 m_2 G_2.$$

But (14) only suggests that a change in one fiscal variable has to be accompanied by a change in at least one of the other four local fiscal variables.

[III] The Local Government' Optimal Behavior in the Decentralized Economy

[3.1]

Suppose that each local government in each region has authority to determine its own fiscal variables (G_i , τ_i) ($i = 1, 2$). Consider the following game.

The first stage: The central government determines the net transfer T .

The second stage: Each local government determines the amount of local public goods G_i and the income tax rate τ_i .

The third stage: Each household in one region decides whether to move to another re-

- 6) In the neoclassical model it always holds that $N_1 = L_1$ and that $N_2 = L_2$. Therefore, under the lump-sum tax Z_i ($i = 1, 2$),

$$U_1 = \alpha \ln (1 - Z_1/N_1) + \beta \ln (Z_1 + T)$$

and

$$U_2 = \alpha \ln (1 - Z_2/N_2) + \beta \ln (Z_2 - T).$$

Then, as is easily proved, given Z_i ($i = 1, 2$) and T , the equilibrium point at which (6) holds is unstable. Unlike Najima (1995), under the assumption of pure local public goods Seoka (2002) derived the neoclassical case that transfer from the overpopulated region to the underpopulated one leads to the *stable* equilibrium with inhabitants in both regions. But at that equilibrium an increase in transfer decreases (the private part of) utility of the household in the whole economy.

- 7) There are two concepts of “the propensity to import”. One is the ratio of the amount of imports to the amount of regional products and another is the ratio of the amount of imports to regional income (the amount of products + transfer received). Without any difference in conclusion I choose the former definition.

gion or not.

The usual backward solution is as follows.

Firstly, given the fiscal variables, (G_1, τ_1) , (G_2, τ_2) and T , the behavior of the household results in the equalization of utility between two regions. That is, (6) holds and determines N_1 and N_2 as the function of all the fiscal variables.

Secondly, given T and the expected values (G_2^e, τ_2^e) in region 2, the local government in region 1 (the local government 1) maximizes utility of the household in its own region with respect to (G_1, τ_1) subject to (6) and (14). This leads to the following results.

$$(15) \quad \tau_1 = \frac{\beta(G_1 - T)}{(\alpha + \beta)G_1 - \beta T}.$$

Since the government 1 expects the government 2 to behave in the similar way,

$$(15)' \quad \tau_2^e = \frac{\beta(G_2^e + T)}{(\alpha + \beta)G_2^e + \beta T}$$

holds. τ_2^e denotes the rate of income tax which the local government 1 expects the local government 2 to impose on inhabitants in region 2. Similarly for G_2^e .

By substituting these equations into (14) we obtain

$$(16) \quad m_1 G_1 - m_2 G_2^e = \frac{\beta}{\alpha + \beta} (1 + m_1 + m_2) T,$$

which gives the maximum of G_1 under a given G_2^e .

Similarly for the local government 2. That is,

$$(16)' \quad m_1 G_1^e - m_2 G_2 = \frac{\beta}{\alpha + \beta} (1 + m_1 + m_2) T.$$

Thus if the expectations of both local governments are to be consistent with each other, the following equation will hold.

$$(17) \quad m_1 G_1 - m_2 G_2 = \frac{\beta}{\alpha + \beta} (1 + m_1 + m_2) T.$$

From (17) we see that, given T , the optimal amount of G_1 is constrained by that of G_2 and vice versa.

How can such an adjustment take place? We can imagine the following adjustment mechanism by an "auctioneer".

Notice that the following relationship always holds.

$$\begin{array}{ccc} & > & < \\ G_1 = G_1^e & \Leftrightarrow & G_2 = G_2^e, \\ & < & > \end{array}$$

where G_2^e is G_2 which is expected by the local government 1. The local government 1 offers G_1 which is determined from (16) under a given G_2^e . Similarly for the local government 2. Suppose, for instance, that initially $G_1 > G_1^e$ and $G_2 < G_2^e$. Then while the local government 1 will lower his G_2^e , the local government 2 will raise his G_1^e . This makes G_1 fall and G_2 rise. Such a process will continue till $G_1 = G_1^e$ and $G_2 = G_2^e$ are achieved (see Appendix).

[3.2]

From (10) and (15) we obtain for the household in region 1

$$(18) \quad U_1 = A + (\alpha + \beta) \ln G_1 - \alpha \ln N_1,$$

where A is a constant. Similarly

$$(18)' \quad U_2 = A + (\alpha + \beta) \ln G_2 - \alpha \ln N_2$$

for the household in region 2. By substituting these equations into (6) we obtain

$$(19) \quad \frac{N_1}{N_2} = \left[\frac{G_1}{G_2} \right]^{(\alpha + \beta)/\alpha}.$$

In particular when $T = 0$, from (17)

$$(20) \quad m_1 G_1 = m_2 G_2.$$

Therefore from (19)

$$(21) \quad \frac{N_1}{N_2} = \left[\frac{G_1}{G_2} \right]^{(\alpha + \beta)/\alpha} = \left[\frac{m_2}{m_1} \right]^{(\alpha + \beta)/\alpha}.$$

On the other hand we obtain the following results from (4), (4)', (15), (15)' and (21).

$$\frac{Y_1/N_1}{Y_2/N_2} = \left[\frac{m_1}{m_2} \right]^{\alpha/\beta}.$$

Thus it holds that

$$(22) \quad \frac{m_1}{m_2} = \frac{G_2}{G_1} = \frac{Y_2}{Y_1} \begin{matrix} > \\ < \end{matrix} 1 \Leftrightarrow \frac{N_2}{N_1} \begin{matrix} > \\ < \end{matrix} 1 \Leftrightarrow \frac{Y_1/N_1}{Y_2/N_2} = \frac{c_1}{c_2} \begin{matrix} > \\ < \end{matrix} 1,$$

which implies that if $m_2 > m_1$, per capita income is larger in region 2 than in region 1 although utility is equal between two regions.

Why is Y_1/N_1 smaller than Y_2/N_2 if $m_2 > m_1$? When $m_1 = m_2$, $c_1 = c_2$, $G_1 = G_2$ and $N_1 = N_2$. Now suppose that m_2 is higher than m_1 . Then the balance of interregional trade without transfer requires that G_1 be larger than G_2 . This tends to make welfare in region 1 larger than that in region 2. Therefore the population tends to move from region 2 to region 1. But this movement does not change output in both regions, which is determined by effective demand. Therefore y_1 and c_1 tend to fall and y_2 and c_2 tend to rise.

We will consider below transfer from a region with high per capita income to a region with low per capita income. But notice that this means transfer from a region with the small population to a region with the large population, which is contrary to Nakajima (1995) and other traditional literature.⁸⁾

[3.3]

From (19)

8) This strangeness will disappear if we divide the economy into the urban area and the rural area. The former has the higher density of population although its absolute number is smaller. Tokyo has ten millions inhabitants while the areas other than Tokyo have one hundred millions inhabitants. Important is that, in equilibrium, Tokyo cannot but have the higher per capita income since it has the higher propensity to import.

$$(23) \quad N_1 = \frac{G_1^{(\alpha+\beta)/\alpha} N}{G_1^{(\alpha+\beta)/\alpha} + G_2^{(\alpha+\beta)/\alpha}}$$

$$(23)' \quad N_2 = \frac{G_2^{(\alpha+\beta)/\alpha} N}{G_1^{(\alpha+\beta)/\alpha} + G_2^{(\alpha+\beta)/\alpha}}$$

Thus (18) and (18)' are reduced to the following equation.

$$(24) \quad U = A + \alpha \ln (G_1^{(\alpha+\beta)/\alpha} + G_2^{(\alpha+\beta)/\alpha}).$$

Therefore welfare depends on

$$(25) \quad V \equiv G_1^{(\alpha+\beta)/\alpha} + G_2^{(\alpha+\beta)/\alpha}$$

Now from (17)

$$(26) \quad dG_1 = \frac{m_2}{m_1 + m_2} dG + \frac{\beta}{\alpha + \beta} \frac{1 + m_1 + m_2}{m_1 + m_2} dT,$$

where $G \equiv G_1 + G_2$. Therefore from (25)

$$(27) \quad dU \Rightarrow (G_1^{\alpha/\beta} - G_2^{\alpha/\beta}) \frac{\beta}{\alpha + \beta} \frac{1 + m_1 + m_2}{m_1 + m_2} dT \\ + \left(\frac{m_2}{m_1 + m_2} G_1^{\beta/\alpha} + \frac{m_1}{m_1 + m_2} G_2^{\beta/\alpha} \right) dG$$

Since $G_1 > G_2$ from (17) under the assumptions that $m_1 < m_2$ and that $T \geq 0$, we obtain the following proposition.

Proposition 2

Suppose that $dG/dT = 0$ and that $T \geq 0$. Then an increase in T (transfer from region 2 to region 1) increases economic welfare in the whole economy iff

$m_2 > m_1$. //

The assumption that the total amount of government expenditures is constant in the face of a change in T may seem to be arbitrary⁹⁾ but this assumption helps to distinguish the problem of the allocation of public expenditures from the problem of their total amount, which we will discuss in the next section.

In relation to Proposition 2 we obtain the following corollaries. Suppose that $T \geq 0$ and that $dG/dT = 0$. Then in the case that $m_2 > m_1$,

Corollary 2-1 public expenditures

(i) $G_1 > G_2$

(ii) *An increase in T always increases G_1 and decreases G_2 by the same amount.*

Corollary 2-2 output

(i) $Y_1 > Y_2$

(ii) *An increase in T increases Y_1 and decreases Y_2 by the same amount.*

Corollary 2-3 the population

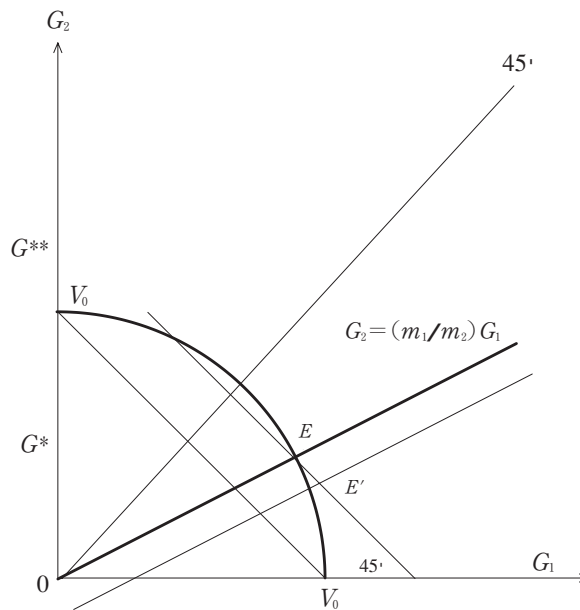
(i) $N_1 > N_2$

(ii) *An increase in T always increases N_1 and decreases N_2 .*

Corollary 2-4 per capita output

9) However, see Appendix below.

Fig. 2



- (i) $y_1 \equiv Y_1/N_1 < y_2 \equiv Y_2/N_2$
- (ii) An increase in T always increases y_2/y_1 .

Corollary 2-5 the rate of income tax

- (i) $\tau_1 \leq \tau_2$ (equality holds if $T = 0$)
- (ii) An increase in T always lowers τ_1 and raises τ_2 .

Corollary 2-6 per capita consumption

- (i) $c_1 < c_2$.
- (ii) An increase in T always increases c_2/c_1 . //

These corollaries show that an increase in T will further enlarge differences in almost all the economic variables between two regions while it increases welfare.

The illustrative explanation of Proposition 2 is as follows.

For simplicity assume that $T=0$ initially. In Fig. 2 the point E is the initial point which is simultaneously on the "indifference curve" (25) (curve V_0V_0) and on the trade balance line $m_1G_1 = m_2G_2$. Notice that the point E is below the 45-degree line since $m_2 > m_1$ and that the indifference curve is concave towards the origin.

Now suppose that transfer from region 2 to region 1 occurs by ΔT . Then the trade balance line shifts downwards and if $dG=0$, the point E moves towards the point E' along the line which has the slope of -45° and passes through the point E . The point E' is outside the initial indifference curve V_0V_0 . Therefore welfare is larger at the point E' than at the point E .

In the above explanation it is important (a) that the indifference curve is concave

towards the origin and (b) that initially G_1 is larger than G_2 . An increase in public expenditures has the double positive effects, the direct effect that it increases public services and the indirect effect that it increases the effective demand and increases the private consumption. These effects give the public expenditures the “increasing returns to scale” property, which makes the “indifference curve” concave toward the origin. Thus If G_1 is larger than G_2 initially and G_1 increases by the same amount as G_2 decreases, welfare will improve.¹⁰⁾

[3.4]

As was mentioned above, welfare is larger, the larger the transfer from region 2 to region 1 is, even if it does not increase the total output in the whole economy. Then what is the optimal transfer for the central government that aims at maximizing welfare ?.

Suppose that initially $T=0$ and that correspondingly $G_1=G_1(0)$ and $G_2=G_2(0)$. $G_1(0)$ and $G_2(0)$ meets the trade-balance condition. That is, $m_1G_1(0)-m_2G_2(0)=0$.

Now when $T>0$, from (9) and (15), employment in region 1 is given by

$$(28) \quad L_1 = \frac{\alpha + \beta}{\beta} G_1 - T.$$

T , in turn, can be represented by G_1 as the solution of the simultaneous equations, (17) and

$$G_1 + G_2 = G_1(0) + G_2(0) \equiv G(0).$$

That is,

$$(29) \quad T = \frac{\alpha + \beta}{\beta(1 + m_1 + m_2)} \{ (m_1 + m_2)G_1 - m_2G(0) \},$$

Substituting (29) into (28) we obtain

$$(30) \quad L_1 = \frac{\alpha + \beta}{\beta} \frac{G_1 + m_2G(0)}{1 + m_1 + m_2}.$$

Similarly

$$(30)' \quad L_2 = \frac{\alpha + \beta}{\beta} \frac{G_2 + m_1G(0)}{1 + m_1 + m_2}.$$

holds.

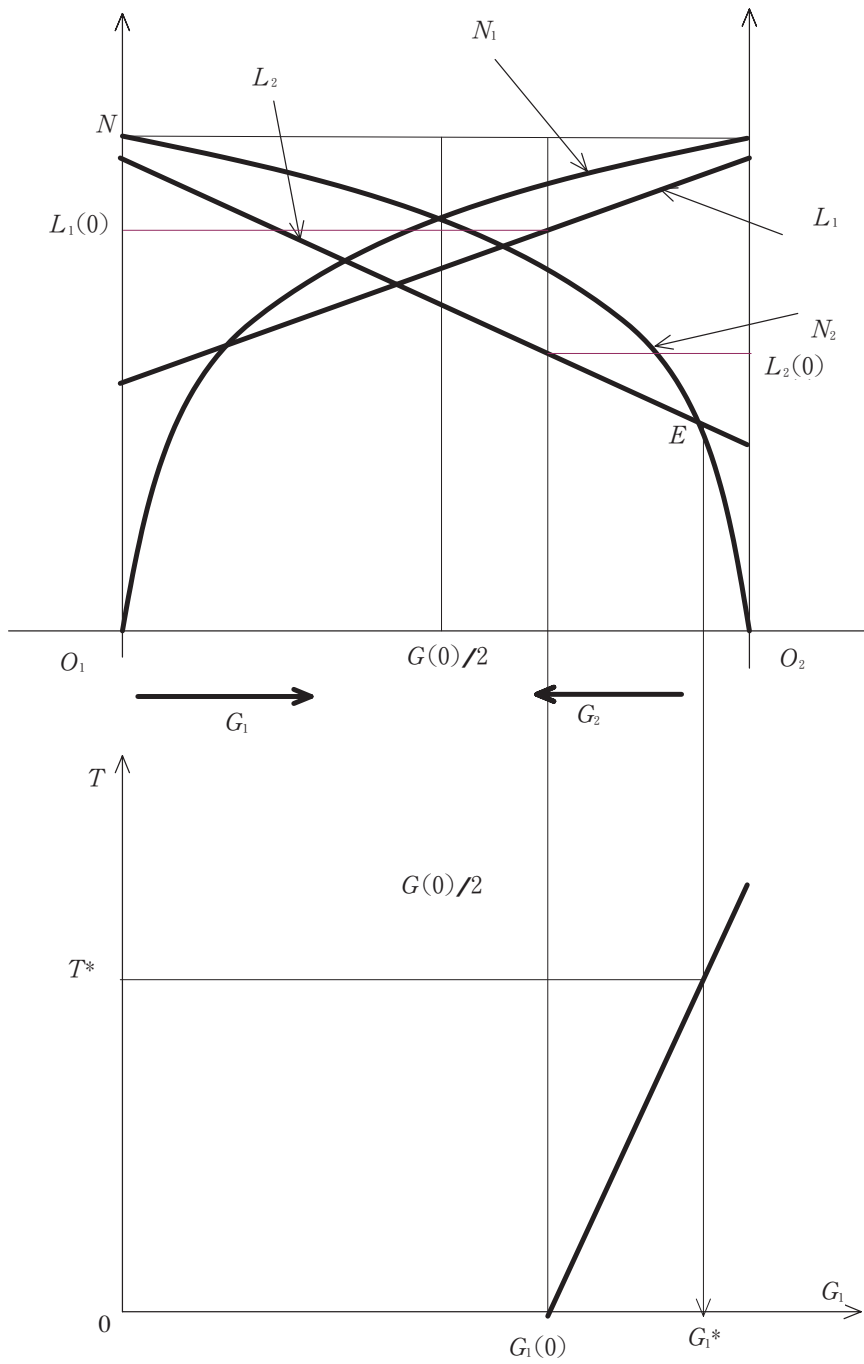
On the other hand N_1 and N_2 are given by (23) and (23)' respectively. Thus since $G_1 > G_2$ and $m_1G_1 > m_2G_2$, it holds that

$$\frac{L_1}{L_2} = \frac{G_1 + m_2G(0)}{G_2 + m_1G(0)} < \frac{G_1}{G_2} < \left[\frac{G_1}{G_2} \right]^{(\alpha + \beta)\alpha} = \frac{N_1}{N_2},$$

which implies that when L_2 is equal to N_2 , N_1 is still larger than L_1 .

10) As is seen in Appendix, if the velocity of adjusting G_i^e ($i = 1, 2$) is equal between two local governments, there is another positive effect of transfer. It is due to the fact that the government expenditures have the larger multiplier effect in region 1 with the smaller propensity to import than in region 2 with the larger propensity to import.

Fig. 3



In the upper part of Fig. 3 G_1 and G_2 are measured from O_1 and O_2 respectively. The distance O_1O_2 is equal to $G(0)$. Suppose that initially $L_1(0) < N_1(0)$ and $L_2(0) < N_2(0)$ under $G_1 = G_1(0)$ and $G_2 = G_2(0)$ and then that as T increases, G_1 increases and G_2 decreases. Finally we will reach the state (the point E) that $L_2 = N_2$ although still $L_1 < N_1$ ¹¹⁾. In the lower part of Fig. 3, (29) is illustrated. T^* which corresponds to the point E shows the optimal transfer in the decentralized economy.

Proposition 3

Suppose that $dG/dT = 0$ in the decentralized economy. Then, even if the central government uses transfer so as to make welfare as large as possible, full employment is achieved only in region 2 while region 1 still remains in the state of underemployment.

[IV] Welfare in the Centralized Economy

[4.1]

Full employment may not be achieved in the decentralized economy for two reasons.

- (a) Since the amount of one local government's expenditures in public goods is constrained by another local government's policy, the total amount of expenditures may not be enough to achieve full employment.
- (b) Even if the transfer can be used by the central government for administrating the behavior of the local governments, it can achieve neither full employment in both regions nor maximal welfare in the whole economy.

These problems are serious in the decentralized economy, as was seen in the previous section. In this section we deal with the centralized economy in order to compare it with the decentralized economy.

Consider the following game.

- (i) The central government determines all the fiscal variables.
- (ii) The households determine which region to live in.

Specifically the central government maximizes U_1 represented by (10) subject to (6), (9), (9)' and (14).

The Lagrangian is

11) Initially $L_1(0) < N_1(0)$ and $L_2(0) < N_2(0)$. Therefore from (23), (23)' (30) and (30)' we obtain

$$N > \frac{\alpha + \beta}{\beta} G(0).$$

Hence it holds that

$$N > \frac{1 + m_1}{1 + m_1 + m_2} \frac{\alpha + \beta}{\beta} G(0).$$

This is precisely the necessary and sufficient condition for the intersection point E of curve N_2 with line L_2 to uniquely exist within the relevant zone. Notice that, as T increases, L_2 decreases but N_2 decreases more sharply to achieve full employment in region 2.

$$L = U_1 + \lambda (U_1 - U_2) + \mu \{T(1 + t_1 m_1 + t_2 m_2) - t_1 m_1 G_1 + t_2 m_2 G_2\} \\ + \theta_1 \{N_1 - t_1 (G_1 - T)\} + \theta_2 \{N_2 - t_2 (G_2 + T)\}$$

Notice that $\theta_1 = 0$ if inequality holds in (9). Similarly for θ_2 .

Now, as is easily proved, whether θ_i ($i = 1, 2$) may vanish or not, the optimal rates of income tax are given by (15) and (15)'. Hence (17) holds. Therefore the above maximizing problem is reduced to maximizing (25) subject to

$$(31) \quad \frac{G_1^{(\alpha+\beta)/\alpha} N}{G_1^{(\alpha+\beta)/\alpha} + G_2^{(\alpha+\beta)/\alpha}} \geq \frac{(\alpha+\beta)}{\beta} \frac{G_1(1+m_2) + m_2 G_2}{1+m_1+m_2}$$

and

$$(31)' \quad \frac{G_2^{(\alpha+\beta)/\alpha} N}{G_1^{(\alpha+\beta)/\alpha} + G_2^{(\alpha+\beta)/\alpha}} \geq \frac{(\alpha+\beta)}{\beta} \frac{G_2(1+m_1) + m_1 G_1}{1+m_1+m_2}.$$

Notice that if equality holds in (31), full employment is achieved in region 1. Similarly for (31)'.

Curve OA_1 in Fig. 4 illustrates the relationship between G_1 and G_2 in the case that equality holds in (31). All the points inside the curve meet the inequality of (31). Similarly curve OA_2 illustrates (31)'. Curve OA_1 and curve OA_2 would be symmetrical with respect to 45° line unless there were the difference between m_1 and m_2 . Let the intersection point between curve OA_1 and 45° line and the intersection point between curve OA_2 and 45° line be (G^*, G^*) and (G^{**}, G^{**}) respectively. Then, as is easily proved, $G^* < G^{**}$ under our assumption that $m_2 > m_1$. Notice also that the vertex of curve OA_1 is below the 45° line.

The solution is feasible if and only if it is within the area enclosed by both curve OA_1 and OA_2 (the shaded area). Thus the optimal point is E, through which the indifference utility curve VV with the largest utility within the shaded area passes. Clearly the point E is below 45° line.

Proposition 4

If the central government can determine all the fiscal variables so as to maximize welfare full employment is achieved in the whole economy. //

Then the amount of government expenditures is larger in region 1 than in region 2 under the assumption that the propensity to import is lower in region 1 than in region 2. This implies that the population is larger in region 1 than in region 2.

[4.2]

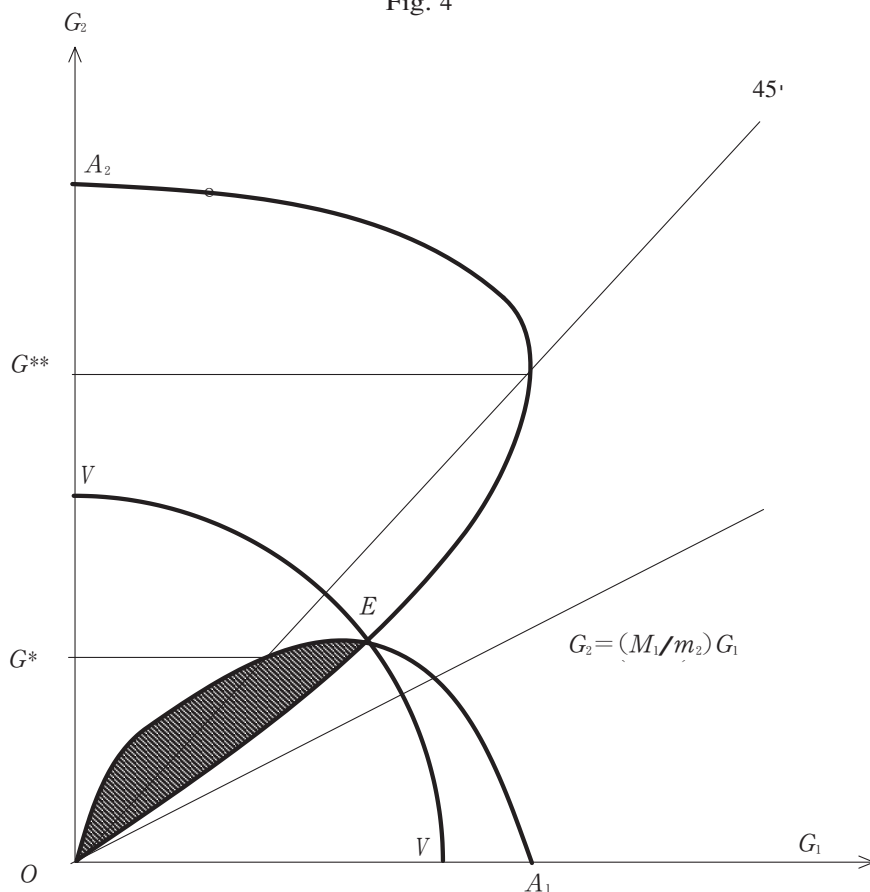
At point E full employment is achieved in both regions. What kind of transfer does this point imply?

We already know that

$$(19) \quad \frac{N_1}{N_2} = \left[\frac{G_1}{G_2} \right]^{(\alpha+\beta)/\alpha}.$$

holds. On the other hand by substituting (15) and (15)' into the right hand sides of (9)

Fig. 4



and (9)', we obtain

$$(32) \quad \frac{L_1}{L_2} = \frac{\alpha G_1 + \beta(G_1 - T)}{\alpha G_2 + \beta(G_2 + T)}$$

In order for full employment to be achieved in both regions it must hold that $N_1/N_2 = L_1/L_2$. Therefore the following relationship holds.

$$(33) \quad \frac{N_1}{N_2} \begin{matrix} > \\ = \\ < \end{matrix} \frac{L_1}{L_2} \Leftrightarrow \left[\frac{G_1}{G_2} \right]^{(\alpha+\beta)/\alpha} \begin{matrix} > \\ = \\ < \end{matrix} \frac{\alpha G_1 + \beta(G_1 - T)}{\alpha G_2 + \beta(G_2 + T)}.$$

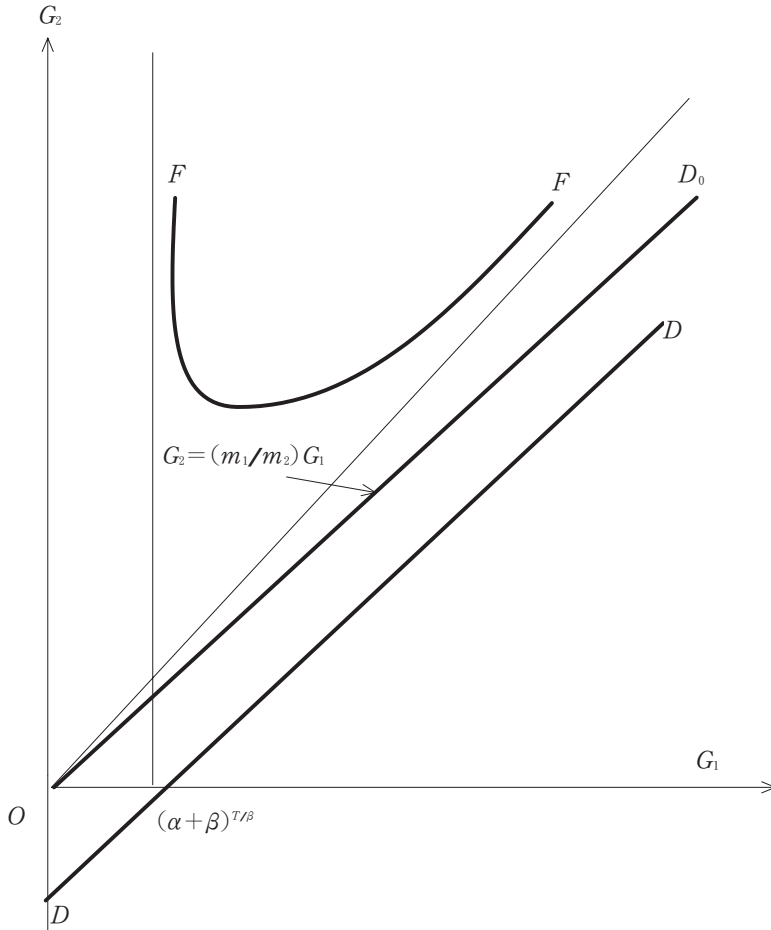
Furthermore by substituting (15) and (15)' into (17), we obtain (17).

Now at first let us suppose that $T=0$. Then (33) is reduced to that

$$\frac{N_1}{N_2} \begin{matrix} > \\ = \\ < \end{matrix} \frac{L_1}{L_2} \Leftrightarrow \frac{G_1}{G_2} \begin{matrix} > \\ = \\ < \end{matrix} 1.$$

However, when $T=0$, (17) is reduced to that $G_1/G_2 = m_2/m_1$ (line OD_0 in Fig. 5).

Fig. 5



Therefore, unless m_1 happens to be equal to m_2 , full employment in both regions can not be achieved. Specifically if $m_2/m_1 > 1$, as has been assumed, $G_1/G_2 > 1$. So $N_1/N_2 > L_1/L_2$. Then even if $L_2 = N_2$ is achieved, $N_1 > L_1$ still holds, which means that full employment cannot be achieved in region 1.

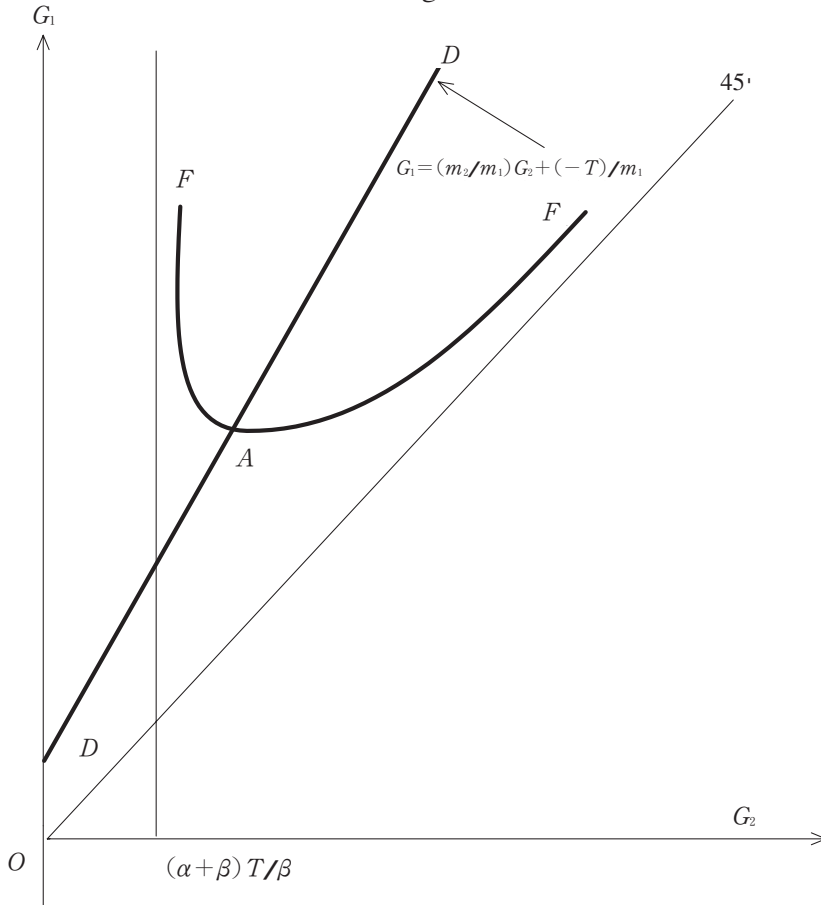
Now suppose that T (transfer from region 2 to region 1) is positive. In Fig. 5 curve FF illustrates the case that equality holds in (33), i.e.

$$(34) \quad \frac{G_2^{(\alpha+\beta)/\alpha}}{(\alpha+\beta)G_2 + \beta T} = \frac{G_1^{(\alpha+\beta)/\alpha}}{(\alpha+\beta)G_1 - \beta T}$$

which is the condition required for full employment in the whole economy.

One asymptote of curve FF is vertical and given by $G_1 = \beta/(\alpha + \beta)$. Another asymptote which curve FF approaches as G_1 increases is 45° line. We can easily prove that curve FF is always above 45° line.

Fig. 6



On the other hand line DD illustrates (17). The G_2 -intercept of this line is negative since T is positive and its slope (m_1/m_2) is smaller than unity. Therefore the curve EE and the line DD never have the intersection point in the first quadrant.

Proposition 5

For achieving the largest welfare there should be transfer from the region with the lower propensity to import to the region with the higher propensity to import. //

Notice that the desired direction of transfer is opposite to that in the case of the decentralized economy. Fig. 6 shows that in the case that $m_2 > m_1$, transfer from region 1 to region 2 (negative T) can achieve full employment. In Fig. 6 G_1 and m_1 are replaced by G_2 and m_2 respectively and vice versa. Specifically curve FF represents

$$(35) \quad \frac{G_2^{(\alpha+\beta)/\alpha}}{(\alpha+\beta)G_2 - \beta(-T)} = \frac{G_1^{(\alpha+\beta)/\alpha}}{(\alpha+\beta)G_1 + \beta(-T)},$$

where $(-T)$ denotes the amount of transfer from region 1 to region 2. On the other hand

line DD represents

$$(36) \quad m_2 G_2 - m_1 G_1 = \frac{\beta}{\alpha + \beta} (-T).$$

In this case curve FF and line DD have always the unique intersection point A in the first quadrant, as is illustrated.

[V] Conclusion

This paper presented a two region model with the local pure public goods in the Keynesian framework and examined how the grant from one region to another influences economic welfare in the whole economy. I obtain the following main results.

(i) The Keynesian model has the equilibrium which has inhabitants in both regions and is stable under the given fiscal variables.

(ii) In the decentralized economy suffering from shortage of effective demand in both regions, the grant from one region with higher propensity to import to another with lower propensity to import improves welfare.

(iii) In the centralized economy full employment in the whole economy can be achieved with maximum welfare. But in this case there should be the grant from the region with lower propensity to import to the region with higher propensity to import.

I admit that this paper has many weak points. Among others I criticize myself about the fact that the propensity to import is regarded as an exogenous variables. In other words giving a micro-foundation to the propensity to import is one of the most important problems that remain unsolved in this paper.

Appendix

We assume in subsection [3.3] that $dG=0$ How plausible is this assumption?

Suppose that T increases by ΔT . Let us assume that the value of G_1 expected by the local government 2 (G_1^e) is revised in the following adaptive way.

$$(a.1) \quad dg_1^e/dv = \gamma_1(G_1(v) - G_1^e(v)).$$

Similarly

$$(a.1)' \quad dG_2^e/dv = \gamma_2(G_2(v) - G_2^e(v)).$$

$G_1(v)$ and $G_2(v)$ are given by the following equations.

$$(a.2) \quad m_1 G_1(v) - m_2 G_2^e(v) = K(T + \Delta T)$$

$$(a.2)' \quad m_1 G_1^e(v) - m_2 G_2(v) = K(T + \Delta T),$$

where

$$K \equiv \frac{\beta}{\alpha + \beta} (1 + m_1 + m_2).$$

$G_1(0)$ and $G_2(0)$ meet

$$m_1 G_1(0) - m_2 G_2(0) = KT.$$

Then by solving these differential equations, we obtain

$$G_1(v) \rightarrow G_1(0) + \frac{\gamma_1 K \Delta T}{(\gamma_1 + \gamma_2) m_1} \equiv \overline{G}_1 \quad (v \rightarrow \infty)$$

and

$$G_2(v) \rightarrow G_2(0) + \frac{\gamma_2 K \Delta T}{(\gamma_1 + \gamma_2) m_2} \equiv \overline{G}_2 \quad (v \rightarrow \infty)$$

Hence

$$\Delta G \equiv \Delta G_1 + \Delta G_2 = \frac{K \Delta T}{\gamma_1 + \gamma_2} \left[\frac{\gamma_1}{m_1} - \frac{\gamma_2}{m_2} \right],$$

where $\Delta G_i \equiv \overline{G}_i - G_i(0)$ ($i=1, 2$). Thus if and only if $m_1/m_2 = \gamma_1/\gamma_2$, we obtain that $\Delta G=0$. If $\gamma_1 = \gamma_2$, however, it will hold that $\Delta G > 0$ under the assumption that $m_2 > m_1$. In this case, welfare improves further by transfer from region 2 to region 1.

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