The scenario on fermion mass generation in electro-weak model including vector-like fermions within USY hypothesis

Katsuichi HIGUCHI (Otemon Gakuin University)

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Abstract

We describe an outline of fermion mass generation in the electroweak gauge model including E6 inspired vector-like fermions in the framework of the hypothesis of Universality Strength for Yukawa couplings (USY). Due to the USY phase breaking the flavor democracy, it is naturally explained that massless fermions have comparatively large masses, and hierarchies for fermion masses are generated in the model.

1. Introduction

The mass hierarchy of fermions is one of the most difficult problems in elementary particle physics. To solve this problem, a lot of hypothesises which constrain Yukawa couplings have been proposed. The hypothesis of flavor democracy in which all Yukawa couplings with each type fermion have the same values is one of them [1,2]. Therefore, some models which are derived from the hypothesis have been analyzed. The model within the hypothesis of Universality Strength for Yukawa couplings (USY) [3] is one of the evolved models in which all Yukawa couplings with each type fermion have the same absolute values, and have different phases respectively in order to reproduce small masses for fermions of the first or the second generation. The Standard Model (SM) within the USY has been analyzed by some researchers. Furthermore, the USY model including vector-like fermions has been studied [4,5,6]. In this paper, we will give an explanation that mass hierarchy is generated naturally and experimental constraints are naturally satisfied by the generated masses of fermions in the above model.

2. The 3×3 toy model

The Yukawa couplings in our model is a combination of the 3×3 mass matrices in the SM.

We demonstrate the diagonalization of the 3×3 mass matrix here.

The genuine democratic 3×3 mass matrix is diagonalized by an orthogonal matric F as below:

$$F^T M F = \frac{\lambda_v v}{3} F^T\ \mathbf{Y} F = \frac{\lambda_v v}{3}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \tag{1}$$

here,

$$F = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 0 & 2/\sqrt{6} & 1/\sqrt{3} \end{bmatrix} \text{ and } \mathbf{Y} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \tag{2}$$

The USY model breaking flavor democracy is proposed in order to generate fermion masses of first and second generation. In the USY model, all Yukawa couplings with each type fermion have the same absolute values and have different phases respectively. Here, we suppose that the only one of the opposite angle ingredients of the third generation has USY phase. The mass matrix is as below:

$$M' = \frac{\lambda_v v}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & e^{i\phi} \end{bmatrix}. \tag{3}$$

Thus, the corresponding eigenvalues and eigenvectors are perturbatively estimated as follows.

$$\text{The eigenvalues : } \lambda \simeq 0,\ \frac{2}{3}i\phi,\ 3$$

$$\text{The eigenvectors : } \mathbf{x} \simeq \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} -1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

The fermion mass of the first generation may be generated from terms of the secondary or higher order because the terms are disregarded.

To summarize so far, the mass of the third generation is produced from the main term in flavor democracy, that of the second generation is derived from the first order of the USY phase breaking flavor democracy, and that of the first generation is generated from the secondary or more higher order of the phase. Then the following hierarchy is automatically reproduced as,

"(The mass of 1st generation) < (The mass of 2nd generation) << (The mass of 3rd generation)" .

The mass matrix M' is diagonalized by unitary matrices, V_L and V_R as

$$V_L^* M' V_R \simeq \frac{\lambda_\nu v}{3}\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{2}{3}i\phi & 0 \\ 0 & 0 & 3 \end{bmatrix}, \tag{4}$$

here

$$V_L = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 0 & \left(2+\frac{2}{3}i\phi\right)/\sqrt{6} & \left(1+\frac{1}{3}i\phi\right)/\sqrt{3} \end{bmatrix} \simeq F \quad \text{and}$$

$$V_R = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 0 & \left(2-\frac{2}{3}i\phi\right)/\sqrt{6} & \left(1-\frac{1}{3}i\phi\right)/\sqrt{3} \end{bmatrix} \simeq F\,. \tag{5}$$

The above diagonalization is barely changed from the genuine democratic case.

3. The model and mass generation

As mentioned in the previous paper [5,7], in our model, there are vector-like SU(2) singlet down-type quarks (D) , vector-like SU(2) doublet charged leptons(E), and SU(2) singlet Higgs inspired E6 GUTs which are not included in the SM. In addition, the three ansatz are imposed for Yukawa couplings in the model.

1. Yukawa couplings of the standard Higgs doublet have the same strength.
2. Yukawa couplings of the singlet Higgs have the same strength.
3. Neutrinos have Majorana masses.

In our past research [4], it is proved that at least two SU(2) singlet down-type quarks are demanded for down-type quark sector to reproduce comparatively large CP violating phase in the CKM matrix. Accordingly, in the USY model in this paper, we also suppose that there exist three generations for each type fermions.

In the following, we comment on diagonalization of mass matrix in genuine flavor democratic case and aspect of generation of fermion masses caused to the USY phase breaking flavor democracy for each four type fermions.

3-1. The group of up-type quarks

The calculation in the second chapter may be directly applied to mass generation up-type quarks. The mass matrix of these quarks is as follows:

$$M_u' = \frac{\lambda_v v}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & \exp(i\phi_{u33}) \end{bmatrix} \tag{6}$$

Then, it is derived from experimental results for m_c and m_t [8] as

$$\phi_{u33} \simeq \frac{9}{2}\frac{m_c}{m_t} \approx 0.03 .$$

Then, m_t becomes to order 1, m_c becomes to the first order of the phase, and m_u becomes to the second or higher order of that. Therefore the mass hierarchy of up-type quarks can be naturally explained.

3-2. The group of down-type quarks and the group of charged leptons

Using the method of our past research [4], the mass matrices of down-type quarks and charged leptons are shown as follows:

$$M_{\mathrm{D}} = \frac{\lambda_\nu v}{3}\begin{bmatrix} \mathbf{Y} & \mathbf{Y} \\ k\mathbf{Y} & k\mathbf{Y} \end{bmatrix} \rightarrow \lambda_\nu v \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k+1 \end{bmatrix} \tag{7}$$

$$M_{\mathrm{E}} = \frac{\lambda_\nu v}{3}\begin{bmatrix} \mathbf{Y} & k\mathbf{Y} \\ \mathbf{Y} & k\mathbf{Y} \end{bmatrix} \rightarrow \lambda_\nu v \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k+1 \end{bmatrix} \tag{8}$$

For these type fermions, the mass hierarchy between the first or second generation and the third generation can be naturally explained. On the other hand, such between the first generation and the second generation cannot be naturally explained.

3-3. The group of neutrinos

In this model, neutrinos have Majorana masses. The 12×12 mass matrix is shown with neutrino column matrix as,

$$\begin{bmatrix} \overline{\nu}_{Li} & \overline{N}\nu_{Li} & \overline{\nu}_{Ri} & \overline{N}_{Ri} \end{bmatrix} \frac{\lambda_\nu v}{3} \begin{bmatrix} 0 & 0 & \mathbf{Y} & k\mathbf{Y} \\ 0 & 0 & \mathbf{Y} & k\mathbf{Y} \\ \mathbf{Y} & \mathbf{Y} & x\mathbf{Y} & 0 \\ k\mathbf{Y} & k\mathbf{Y} & 0 & 0 \end{bmatrix} \begin{bmatrix} \nu_{Li} \\ N_{Li} \\ \nu_{Ri} \\ N_{Ri} \end{bmatrix}, \tag{9}$$

where ν_{Li} is left-handed SU(2) doublet ordinary neutrino, N_{Ri} is left-handed SU(2) doublet vector-like neutrino, ν_{RLi} is right-handed SU(2) singlet ordinary neutrino, N_{Ri} is right-handed SU(2) doublet vector-like neutrino, and i=1,2,3. The above mass matrix is diagonalized as follows:

$$\frac{\lambda_\nu v}{3}\begin{bmatrix} 0 & 0 & \mathbf{Y} & k\mathbf{Y} \\ 0 & 0 & \mathbf{Y} & k\mathbf{Y} \\ \mathbf{Y} & \mathbf{Y} & x\mathbf{Y} & 0 \\ k\mathbf{Y} & k\mathbf{Y} & 0 & 0 \end{bmatrix} \rightarrow \lambda_\nu v \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&\sqrt{2}k&0&0 \\ 0&0&0&0&0&0&0&0&0&0&-\sqrt{2}k&0 \\ 0&0&0&0&0&0&0&0&0&0&0&x \end{bmatrix} \quad (10)$$

If the USY phase breaking flavor democracy is added to the three diagonal elements generating three large eigenvalues, three additional eigenvalues which are derived from the first order of the phase are generated. In sum, there are three large eigenvalues, comparatively large eigenvalues, and six small eigenvalues. The appearance of the above discussion is shown as

$$\lambda_\nu v \begin{bmatrix} \delta&0&0&0&0&0&0&0&0&0&0&0 \\ 0&\delta&0&0&0&0&0&0&0&0&0&0 \\ 0&0&\delta&0&0&0&0&0&0&0&0&0 \\ 0&0&0&\sim k\phi&0&0&0&0&0&0&0&0 \\ 0&0&0&0&\sim k\phi&0&0&0&0&0&0&0 \\ 0&0&0&0&0&\sim x\phi&0&0&0&0&0&0 \\ 0&0&0&0&0&0&\delta&0&0&0&0&0 \\ 0&0&0&0&0&0&0&\delta&0&0&0&0 \\ 0&0&0&0&0&0&0&0&\delta&0&0&0 \\ 0&0&0&0&0&0&0&0&0&\sqrt{2}k&0&0 \\ 0&0&0&0&0&0&0&0&0&0&-\sqrt{2}k&0 \\ 0&0&0&0&0&0&0&0&0&0&0&x \end{bmatrix}. \quad (11)$$

There exist nine SU(2) doublet neutrinos in the model. All of them interact with Z boson. On the other hand, three SU(2) singlet neutrinos do not interact with Z boson. Consequently, if the three comparatively large masses caused by the USY phases are larger than the energy of present experimental upper limit, it is naturally explained not only that there is mass hierarchy in each type fermion, but also that all fermion masses derived from this model satisfy experimental constraint of Z boson decay which suggests that there exist only three light neutrinos.

4. Summary

In this paper, we describe the scenario about generating fermion masses in the comprehensive USY model including vector-like fermions inspired by E6 GUTs. In particular, it becomes clear that the hierarchy of three up-type quarks or twelve neutrinos is naturally reproduced, and that the derived masses may satisfy experimental constraints naturally.

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