# Evaluating Political System by Using the Median Voter Theorem*

KATSUYOSHI OKUI**

**Abstract**

The aim of this paper is to evaluate a political system by using the Median Voter Theorem. We measure a tax rate which is most preferred by the median voter and compare it with an actual tax rate. And we take the difference between them as a degree of democracy and measure it. We do the above operations for each case : Japan and the U. S. A. We get the results as follows. First, concerning the preferred tax rate by the median voter, the tax rate is lower in Japan than in the U. S. A. Second, both in Japan and in the U. S. A., the tax rate preferred by the median voter is lower than the actual tax rate. It means that governments are bigger than the majority expect in both countries. And third, judging from our degree of democracy, the U. S. A. is more democratic than Japan.

**Key words** : median voter theorem, political system, democracy, Japan and the U. S. A., income inequality

---

* The author would like to thank participants of the workshops : Otemon Gakuin Daigaku Keizai Gakkai (Osaka, Japan, December 1995) and Kansai Seiji-keizai Kenkyukai (Hyogo, Japan, December 1995) and of the 1997 annual meeting of Public Choice Society (San Francisco, U. S. A., March 1997) for valuable comments. And I would like to thank Dr. Gebhard Kirchgassner for reading the manuscript and making a number of helpful suggestions. But of course I am solely responsible for any errors.

** Faculty of Economics, Otemon Gakuin University 2 – 1 – 15, Nishiai, Ibaraki-shi, Osaka-fu, 567 – 8502, Japan. (e-mail : okui@res.otemon.ac.jp)

## 1 Introduction

The aim of this paper is to evaluate a political system by using the Median Voter Theorem. The Median Voter Theorem means that if a social decision is made by a democratic procedure like direct voting, this decision is identical with the one which the median voter makes. By using the theorem, we can predict a social decision under democratic society. However, if we consider the theorem from a different point of view, this predicted social decision can be considered a criterion examining the degree of democratic procedure when a social decision is made.

That is, the predicted social decision by using the Median Voter Theorem can be considered an ideal one which realizes direct democracy. And the difference between the predicted social decision and an actual social decision can be considered as the difference between ideal democracy and actual democracy. Hence the degree of difference between the predicted social decision and the actual social one can be considered as showing to what degree the society realizes ideal democracy.

For this point of view, this paper compares a tax burden rate which is predicted in the case when the Median Voter Theorem holds with an actual tax burden rate. Especially by taking the cases of Japan and the U. S. A, we will make a comparison of the political system of Japan and the U. S. A.

There are some prior works which looked into whether the predicted social decision by using the Median Voter Theorem is identical with the actual social decision or not, and which made a comparison of political system by using the Median Voter Theorem. Inman (1978) investigated whether the predicted public expenditure by using the Median voter Theorem is identical with the actual one. Pommerehne (1978) took the public expenditure of different political systems, such as direct democracy and representative democracy, and examined under what political system the Median Voter Theorem fits well. Difference between these prior works and my paper is as follows. First, we take a tax rate for the purpose of investigating whether the predicted social decision is identical with the actual social one. Second, we largely take inequality of income distribution into consideration. Recently, there have appeared many works looking into the relationship between political process and income distribution. For example, Alesina = Rodrik (1994), Perotti Roberto (1993, 1994) and Persson = Tabellini (1994). This paper takes advantage of their way of thinking. Third, we introduce a degree of democracy and try to measure to what degree the society realizes ideal democracy. Our way of

measuring may have difficulties. But it is different from the way of Pommerehne and is considered as a new trial to evaluate political system objectively.

The plan of the paper is as follows. Section 2 presents a model which enables us to measure a predicted tax rate. This model is largely debted to Roberts (1977), Meltzer and Richard (1981). By using the model, section 3 measures a predicted tax burden rate and compares it with an actual one. We apply this analysis to the cases of Japan and the U. S. A. and make comparisons between the political system of Japan and of the U. S. A. Section 4 is the conclusion.

## 2 Model

Public finance can be fundamentally considered as a system in which the richer a person is, the more tax he pays and the poorer a person is, the more subsidy he receives.[1] Hence, we can simply assume that the public finance system functions are as follows: incomes of all members of society are taxed proportionately and all collected incomes are distributed equally to them.[2] Assuming that $Y_i$ is individual $i$'s pre-income tax, and $T_i$ is the amount of a tax that individual $i$ pays, and that $t$ is tax rate. The amount of tax revenue that individual $i$ pays is

$$T_i = tY_i. \tag{1}$$

Now we assume that the total amount of tax revenue that all members of the society paid is used up as subsidy to the members. Hence, the total amount of tax revenue is equal to the total amount of subsidy. Since the total amount of subsidy is distributed equally to all members, assuming that the amount of subsidy per member is $\alpha$ and the number of the society is $N$,

1) In our following discussion, the function of government is only to redistribute income. Another various functions of government, especially provision of public goods, are neglected. It may bias our judgment when we explain the empirical results afterwards. This point needs further consideration.

2) Validity of this assumption may be open to question. But Pechman and Okuer (1974) and Pechman (1985) showed that the U. S. tax system is essentially proportional. Pechman and Kaizuka (1976) mentioned that the Japanese tax system is only mildly progressive and has little effect on the relative distribution of income. Therefore we can say that this assumption is not far from the truth.

$$\alpha = \frac{\sum_{n=0}^{n=N} T_i}{N},$$
$$= \frac{t\sum_{n=0}^{n=N} Y_i}{N},$$
$$= t\bar{Y}, \tag{2}$$

where-implies mean. Hence $i$'s disposal income $Y_i^p$ is as follows,

$$Y_i^p = (1-t)Y_i + \alpha. \tag{3}$$

Next we consider each individual $i$'s maximization problem. Assuming that the amount of labour supply produces the same amount of income,

$$L_i = Y_i.$$

Then individual $i$ chooses $Y_i$ so as to maximize $U(Y_i^p, L_i)$. If the number of members of the society is sufficiently large, the first-order condition of maximization is

$$(1-t)\frac{\partial U_i}{\partial Y_i^p} + \frac{\partial U_i}{\partial Y_i} = 0. \tag{4}$$

Individual $i$ chooses $Y_i$ so as to hold equation (4) for any t.

Then next we consider an optimal $t$ for individual $i$. Totally differentiating the utility function,

$$dU_i = dY_i\left\{(1-t)\frac{\partial U_i}{\partial Y_i^p} + \frac{\partial U_i}{\partial Y_i}\right\} + \frac{\partial U_i}{\partial Y_i^p}(d\alpha - Y_i dt). \tag{5}$$

Substituting equation (4) into (5), we get

$$dU_i = \frac{\partial U_i}{\partial Y_i^p}(d\alpha - Y_i dt). \tag{6}$$

Differentiating equation (2) with respect to $t$,

$$\frac{d\alpha}{dt} = \bar{Y} + t\frac{d\bar{Y}}{dt}. \tag{7}$$

Substituting (7) into (6), we get

$$\frac{dU}{dt} = \frac{\partial U_i}{\partial Y_i^p}(\bar{Y} + t\frac{d\bar{Y}}{dt} - Y_i). \tag{8}$$

Hence, individual $i$'s most preferred tax rate is the one that satisfies

$$\bar{Y} + t\frac{d\bar{Y}}{dt} - Y_i = 0. \tag{9}$$

Since the tax rate decided as a social decision is the one that is most preferred by a decisive voter, the tax rate that is decided through the social decision process satisfies

$$\bar{Y} + t\frac{d\bar{Y}}{dt} - Y_d = 0 \tag{10}$$

where $Y_d$ is the income of a decisive voter.

Then who is a decisive voter? According to the Median Voter Theorem, a decisive voter is a median voter. That is, it is the voter whose preferred tax rate is median over all preferred tax rates of all members of the society. However since it is difficult to know the median preferred tax rate, we have to estimate it. In order to estimate it, it is necessary for us to add one assumption. This assumption is the Hierarchical Adherence which was introduced by Roberts (1977). Under this assumption, a decisive voter becomes one which has a median income. Thus if the assumption holds, we can estimate the median preferred tax rate by using income data.

Hierarchical Adherence:

For all $i, j$: either $Y_i(t) \geq Y_j(t)$ for all $t$, or $Y_i(t) \leq Y_j(t)$ for all $t$, where $Y_i(t)$ is the pre-tax income that i chooses to obtain under the tax schedule.

The Hierarchical Adherence states that there exists an ordering of individuals such that pre-tax income is monotonically increasing irrespective of the tax schedule which is in operation. This assumption assures that the voter with a median income becomes the one with a median preferred tax rate.[3)]

Applying the Median Voter Theorem, the tax rate decided as a social decision is the one with a median preferred tax rate. Now under the assumption of the Hierarchical Adherence, the voter who has a median preferred tax is identical with the one who has a median income. Hence in order to estimate the median preferred tax rate, we have only to know a tax rate at which the person with a median income

3) A strict proof is given by Roberts (1977).

maximizes his utility. Hence, the tax rate $t_m$ which is most preferred by the person with a median income satisfies

$$\bar{Y} + t_m \frac{d\bar{Y}}{dt} - Y_m = 0, \tag{11}$$

where $Y_m$ is median income.

In order to calculate a value of $t_m$, we have to decide the concrete form of $\bar{Y}(t)$. Hence we suppose that $\bar{Y}(t)$ has the form as follows.

$$\bar{Y} = k(1-t)^{\beta} \exp\ (r\ time), \tag{12}$$

where $k$, $r$ is a constant parameter and *time* is time trend. This function form means that the elasticity of mean production (income) in response to the fraction of earned income retained is constant $\beta$ and that the trend of economic growth rate is constant $r$. Then

$$\frac{d\bar{Y}}{dt} = -\frac{\beta}{1-t}\bar{Y}. \tag{13}$$

Substituting (13) into (11) and putting $\delta = \frac{Y_m}{\bar{Y}}$

$$t_m = \frac{1-\delta}{\beta + 1 - \delta}. \tag{14}$$

The meaning of $\delta\,(=\frac{Y_m}{\bar{Y}})$ is as follows. As long as $Y_m < \bar{Y}$, above 50 percent of members have a income below a mean income. This situation can be taken to show that there is inequality of income among people. The bigger the difference between $Y_m$ and $\bar{Y}$ is, the more members there are with an income below a mean income. Hence as the value of $\delta$ becomes small, inequality of income becomes large. The increase of $\delta$ means the improvement of equality.

Obviously from equation (14),

$$\frac{dt_m}{d\beta} < 0. \tag{15}$$

And from equation (14) we can get

$$\frac{dt_m}{d\delta} = \frac{-\beta}{(\beta+1-\delta)^2} < 0. \tag{16}$$

(15) means that the bigger the tax response of people, the smaller the tax preferred

by the median voter. And (16) means that the increase of equality reduces the level of the tax preferred by the median voter.

## 3 Empirical Results

At this section we estimate the $t_m$ for Japan and for the U. S. A. In order to do so, we estimate the function $\bar{Y}(t)$ first. Assuming that the function form is like equation (12), we estimate the parameters of it by using regression analysis.

But in this case, there is some doubt about whether $t$ is an exogenious variable. Hence in order to remove the simultaneous bias, we use two stage least squares.[4)]

Moreover, in estimating parameters, we add a lagged $\bar{Y}$ and an Oil Shock dummy variable to the equation (12). The reason why we add a lagged $\bar{Y}$ is that people adjust their output in response to not only the tax rate of the present period but also that of the past period. Hence we assume the pattern of Koyck's lag in examining the effect of tax rate on output (income) and introduce a lagged $\bar{Y}$ to the equation (12).

---

4) In order to use two stage least squarers, we have to estimate $t$ of a reduced form. When we estimate $t$ of a reduced form, we use the following variables as explanatory ones. Government surplus-nominal GDP ratio(G), over 65 year-old population ratio(Z), time trend (time), and oil shock dummy(D). We use Maximum Likelihood with Grid Search Method in order to remove the effect of autocorrelation. Estimated period is 1966 – 90. The results are as follows;

Japan

$$t = 0.164 + 0.177\,G + 0.0382\,Z + 0.00580\,\text{time} - 0.000110\,D \quad \bar{R}^2 = 0.948$$
$$(2.568)\quad(1.662)\quad(0.0345)\quad(2.119)\quad(-0.016)\quad D.W. = 1.649,\ N = 25$$

U. S. A.

$$t = 0.282 + 0.313\,G - 0.0679\,Z + 0.00131\,\text{time} + 0.00490\,D \quad \bar{R}^2 = 0.7789$$
$$(18.983)\quad(2.282)\quad(-0.422)\quad(2.901)\quad(0.831)\quad D.W. = 1.919,\ N = 25$$

Number in parentheses are $t$ values.

t : tax burden rate. Tax burden rate data of both countries are from Revenue Statistics of OECD member countries.

G : government surplus-nominal GDP ratio. Government surplus and nominal GDP data of both countries are from International Financial Statistics Yearbook.

Z : over 65 year-old population ratio. Japanese data are from Demographic Yearbook. Because these data are published only every 5 years, we estimated ones of the rest of the years. The U. S. A. data are from American Report of President.

time : time trend

D : dummy variable putting 0 until 1973 and 1 after 1974.

In this case, estimated $\beta$ can be considered as a short run elasticity of mean production in response to the franction of earned income retained. The reason why we add an Oil Shock dummy is that we would like to take account of the effect of the structual change of the production environment. That is, an estimated equation is as follows;

$$\log \bar{Y} = const + \beta \log (1-t) + r\ time + \delta \log \bar{Y}_{-1} + \eta D. \tag{17}$$

$\bar{Y}$: Real GDP per capita;

$\bar{Y}_{-1}$: Real GDP per capita of the one period ago;

$t$: tax burden rate

*time*: time trend

$D$: Oil schock dummy variable putting 0 until 1973 and 1 after 1974

The result of it is shown in Table 1.

The result shows that the Japanese elasticity of mean production (income) in response to the fraction of earned income retained is much higher than the U. S. A. That is, the effect that an increase of tax has on the reduction of GDP is bigger in Japan than in the U. S. A.

Next we compare the value of $\delta$. According to Table 2, $\delta$ of Japan is bigger than that of U. S. A. This means that Japan has more equality of income than the U. S. A., as was explained before.

Table 1 Estimation result for log $\bar{Y}_t$

[redacted]

Table 2 Comparison of $\delta$

[redacted]

Table 3 Comparison of $t_m$

[redacted]

Table 4 Calculated $t_m$ and actual tax rate

[redacted]

Since we got the values of $\beta$ and $\delta$, now we can calculate $t_m$ by using these values and equation (14). The results of these are shown in Table 3. It shows that tax preferred by the median voter in Japan is lower than in the U. S. A. The reason is as follows.

First, since the Japanese respond more strongly to tax change and reduce more labour supply in response to tax increase, $\beta$ of Japan is higher than that of the U. S. A. Second, Japan has more equality of earned income and has a higher $\delta$ than the U. S. A. These factors make the $t_m$ of Japan lower than that of the U. S. A.

Then we compare the calculated $t_m$ with an actual tax rate for Japan and for the U. S. A. We take a tax burden rate of 1990 as an actual tax rate. Summary of the results appears in Table 4.

Both in Japan and in the U. S. A., the tax rate preferred by the median voter is lower than the actual tax rate. This means that although many people prefer to have a lower tax rate and a policy of redistribution on a smaller scale, a high tax rate is put into practice in both countries. It follows from this that governments are bigger than the majority expect in both countries.

Finally we investigate the difference between the tax rate preferred by median voter and the actual tax rate. As was mentioned before, its difference can be considered as showing to what degree the society realizes ideal democracy. In brief

the smaller the difference is, the more democratic this society is. That is,

$$\text{Degree of Democracy} = | t_m - t | .$$

The result appears in Table 5.

It shows that the degree of democracy of the U. S. A. is small compared to that of Japan. It means that the U. S. A. is more democratic than Japan.[5] In conclusion, judging from our estimation of the degree of democracy, the U. S. A. is more democratic than Japan in the sense that the tax rate which more people wish is realized.

Table 5 Degree of Democracy

[redacted]

## 4 Conclusion

In this paper we estimate a tax rate which is most preferred by the median voter and compare it with an actual tax rate. In so doing, we try to evaluate a political system of a country. We investigate the cases of Japan and the U. S. A. From our investigation we arrive at the conclusion as follows.

First, concerning the preferred tax rate of a median voter, the tax rate is lower in Japan than in the U. S. A. In our model there are two reasons for this result. One reason is that because Japanese respond more strongly to tax change and reduce more labour supply in response to tax increase, they dislike a heavier tax rate. The other reason is that because Japan has more equality of income than the U. S. A., the Japanese people do not want redistribution policy so much as the U. S. A. people. These two reasons make $t_m$ of Japan lower than that of the U. S. A.

Second, both in Japan and in the U. S. A., the tax rate preferred by the median

5) When we judge the result of measurement of degree of democracy, the following question arises: how do we consider the case where a dictator adopts the policy which the median vote prefers? In this case, is the country democratic? In our paper, we consider that the country is democratic in the sense that the dictator meets the expectation of the majority of the country.

The reason why such a problem arises seems that we measure degree of democracy not taking account of frequency that the actual tax rate coincides with the median tax rate but only focusing on the distance between the actual tax rate and the median tax rate. Taking account of frequency that the actual tax rate coincides with the median tax rate seems to be a research subject which should be further pursued.

voter is lower than the actual tax rate. This result means that although many people prefer to have a lower tax rate and a policy of redistribution on a smaller scale, a high tax rate is put into practice in both countries. It follows from this that governments are bigger than the majority expect in both countries.

Third, according to our measurement of the index of degree of democracy, the U. S. A. is more democratic than Japan. In other words, the U. S. A. is democratic compared to Japan in the sense that the tax rate which more people wish is put into practice.

**Reference**

Alesina, A. and Rodrik, D. (1994). Distributive Politics and Economic Growth, *Quarterly Journal of Economics* 109 (May), 467 – 490.

Bergstrom, T. C. and Goodman, R. P. (1973). Private Demands for Public Goods, *American Economic Review* 63(3), 280 – 296.

Inman, R. P. (1978). Testing political economy's 'as if' proposition : is the median income voter really decisive ?, *Public Choice* 33(4), 45 – 65.

Meltzer, A. H. and Richard, S. F. (1981). A Rational Theory of the Size of Government, *Journal of Political Economy* 89(5), 914 – 927.

Mueller, D. C. (1989). *Public Choice II*, Cambridge : Cambridge University Press.

Pechman, J. A. and Okuer B. A. (1974). *Who Bears the Tax Burden ?*, Washington D. C. : The Brooking Institution.

Pechman, J. A. and Kaizuka K. (1976). Taxation. In H. Patrick and H. Rosovsky (Eds.), *Asia's New Giant*, 317 – 382, Washington D. C. : The Brooking Institution.

Pechman, J. A. (1985). *Who Paid the Taxes*, 1966 – 85, Washington D. C. : The Brooking Institution.

Perotti, R. (1993). Political Equilibrium, Income Distribution and Growth, *Review of Economic Studies* 60(4), 755 – 776.

Perotti, R. (1994). Income Distribution and Investment, *European Economic Review* 38, 827 – 835.

Persson, T. and Tabellini, G. (1994). Is Inequality Harmful for Growth ?, *American Economic Review* 84(3), 600 – 621.

Pommerehne, W. W. (1978). Institutional Approaches to Public Expenditure : Empirical Evidence from Swiss Municipalities, *Journal of Public Economics* 9 (April), 163 – 201.

Roberts, K. W. S. (1977). Voting Over Income Tax Schedules, *Journal of Public Economics* 8 (December), 329 – 340.