

# RELIABILITY ANALYSIS FOR A COMPLEX SYSTEM OF DISSIMILAR UNITS

HIROSHI NAKAMICHI

## Abstract

This paper deals with a complex system of dissimilar units including the 2-out-of- $m$  subsystem. We consider two repair disciplines, and under each repair discipline the stationary availability and the mean time to system failure are derived in the exact expression. The approximations for the above formulas are also obtained.

## 1. Introduction

In the previous paper [5], we have considered a system composed of two subsystems connected in series, one of which consists of an active unit and a warm standby and the other consists of several different units connected in tandem. Many studies have been made for multiple-unit systems with repair. Especially, Gaver has discussed systems composed of two identical units [3] and those composed of two dissimilar units [4] with exponential failure and general repair.

As a natural generalization of the above model [5], it may be thought of a model in which the paralleled subsystem consists of three or more dissimilar units. The problem, however, becomes too complex by the condition "dissimilar", because the possible states of the system are too cumbersome to solve the corresponding differential equation. Hence, in this paper we discuss a complex system composed of two series subsystems. One consists of  $m$  dissimilar repairable units treated as the 2-out-of- $m$  system, while the other consists of several different repairable units connected in tandem. We find the long-run availabilities under the two kind of repair disciplines and the mean time to system failure for the system with the exponential failure and the general repair distribution. The approximations for these formulas are also obtained. These results would be useful in practice indeed.

In section 3, we analyze the system availability. The long-run avail-

labilities for the system are derived in the exact expressions from the differential equations of the renewal type, and those approximate formulas are obtained. In section 4, time to system failure is discussed, and the mean time to system failure is derived in the exact and the approximate form in the similar way. In the final section, some numerical examples are given to make a comparison between the exact value and the approximation of the system availability and MTSF.

## 2. Models

We consider a system composed of two subsystems  $S_1$  and  $S_2$  connected in series. Subsystem  $S_1$  consists of  $m$  dissimilar units  $A_1, A_2, \dots, A_m$  connected in parallel, while subsystem  $S_2$  consists of  $n$  different units  $B_1, B_2, \dots, B_n$  connected in tandem. The failure rate of each unit is assumed to be constant, and the failure unit is repaired by a single service channel.

Denote by  $\lambda_j > 0$  the failure rate of unit  $A_j (j=1, 2, \dots, m)$ , and we define  $\lambda \equiv \sum_{j=1}^m \lambda_j$  and  $\lambda_j^* \equiv \lambda - \lambda_j$ . Similarly,  $\nu_k > 0$  denotes the failure rate of unit  $B_k (k=1, 2, \dots, n)$ , and let  $\nu \equiv \sum_{k=1}^n \nu_k$ . We assume that the repair time of unit  $A_j$  obeys an arbitrary distribution function  $F_j(x)$ , i.e., its pdf is

$$f_j(x) = \mu_j(x)[1 - F_j(x)],$$

where  $\mu_j(x)$  is referred to as the repair rate of unit  $A_j$ . Similarly, the repair time of unit  $B_k$  obeys an arbitrary distribution function  $G_k(x)$  and its pdf is  $g_k(x) = \gamma_k(x)[1 - G_k(x)]$ , where  $\gamma_k(x)$  is the repair rate of unit  $B_k$ .

In this paper, we assume that the operation of each unit is fully restored upon repair.

We consider a subsystem  $S_1$  as a 2-out-of- $m$  system assumed to be down when two or more units in the system fail. Therefore, the system fails either when two or more units in  $S_1$  are simultaneously in the failure state or when failure of any unit in  $S_2$  occurs. While the system is in the failure state, the failure rate of each good unit is zero. If any unit in  $S_2$  fails while a failure unit in  $S_1$  is being repaired, it is repaired without delay interrupting the repair of unit in  $S_1$ . Then, we adopt the following repair disciplines;

- 1) Preemptive repeat repair discipline and
- 2) Preemptive resume repair discipline.

We also assume that the reliability of detecting and switching is one and time to switch over is neglected.

### 3. Availability

#### 3.1 Differential equations

Throughout this paper we use the indices  $i$  and  $j$  for unit in  $S_1$ , both of which will always go from 1 to  $m$  and the index  $k$  for unit in  $S_2$  which will also always go from 1 to  $n$ . These indices are used for state probability, summation and so on.

First, we introduce the following notations;

$p_0(t)$ : probability that at time  $t$  all of units  $A_1, \dots, A_m$  and  $B_1, \dots, B_n$  are operating.

$p_j(t, x)dx$ : probability that, at time  $t$  (1) the system is operating, and (2) unit  $A_j$  is in failure and its elapsed repair time (e.r.t.) lies between  $x$  and  $x+dx$ .

$p_{ij}(t, x)dx$ : probability that, at time  $t$  (1) the system is down due to failure of units  $A_i$  and  $A_j$ , (2) unit  $A_i$  is waiting for repair, and (3) unit  $A_j$  is under repair and its e.r.t. lies between  $x$  and  $x+dx$ . ( $i \neq j$ )

$q_k(t, x)dx$ : probability that, at time  $t$  the system is down due to failure of unit  $B_k$  and its e.r.t. lies between  $x$  and  $x+dx$ .

$q_{jk}(t, x)dx$ : probability that, at time  $t$  (1) the system is down due to failure of units  $A_j$  and  $B_k$ , (2) unit  $A_j$  is waiting for repair, and (3) unit  $B_k$  is under repair and its e.r.t. lies between  $x$  and  $x+dx$ .

$q_{jk}(t, x, y)dxdy$ : probability that, at time  $t$  (1) the system is down due to failure of units  $A_j$  and  $B_k$ , (2) unit  $B_k$  is under repair and its e.r.t. lies between  $y$  and  $y+dy$ , and (3) unit  $A_j$  is waiting for repair but  $A_j$  had failed at time  $t-y$  and its e.r.t. lies between  $x$  and  $x+dx$ .

$\delta$ : repair discipline index, i.e.,  $\delta=1$  if the repair discipline is preemptive repeat and  $\delta=0$  if preemptive resume.

Viewing the nature of the system, we obtain the following set of

differential equations ;

$$\begin{aligned}
(1) \quad & \left[ \frac{d}{dt} + \lambda + \nu \right] p_0(t) = \sum_j \int_0^t p_j(t, x) \mu_j(x) dx + \sum_k \int_0^t q_k(t, x) \gamma_k(x) dx, \\
(2) \quad & \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_j^* + \nu + \mu_j(x) \right] p_j(t, x) = (1 - \delta) \sum_k \int_0^{t-x} q_{jk}(t, x, y) \gamma_k(y) dy, \\
(3) \quad & \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_j(x) \right] p_{ij}(t, x) = \lambda_i p_j(t, x) \quad (i \neq j), \\
(4) \quad & \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \gamma_k(x) \right] q_k(t, x) = 0, \\
(5) \quad & \delta \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \gamma_k(x) \right] q_{jk}(t, x) + (1 - \delta) \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \gamma_k(y) \right] \\
& \quad \cdot q_{jk}(t, x, y) = 0.
\end{aligned}$$

Equations (1)~(5) are solved subject to the following boundary conditions (6)~(9) and initial condition (10).

$$\begin{aligned}
(6) \quad & p_j(t, 0) = \lambda_j p_0(t) + \sum_{i \neq j} \int_0^t p_{ji}(t, x) \mu_i(x) dx + \delta \sum_k \int_0^t q_{jk}(t, x) \gamma_k(x) dx, \\
(7) \quad & p_{ij}(t, 0) = 0 \quad (i \neq j), \\
(8) \quad & q_k(t, 0) = \nu_k p_0(t), \\
(9) \quad & \delta q_{jk}(t, 0) + (1 - \delta) q_{jk}(t, x, 0) = \nu_k p_j(t), \\
(10) \quad & p_0(0) = 1.
\end{aligned}$$

### 3. 2 Solutions

#### (i) Preemptive repeat case

In this case we set  $\delta=1$  in the equations of 3.1. Applying the Laplace transform (L. T.) to (1)~(9) with respect to  $t$  under the initial condition (10) in order to solve the equations, we have

$$\begin{aligned}
(11) \quad & [s + \lambda + \nu] \bar{p}_0(s) = 1 + \sum_j \int_0^\infty \bar{p}_j(s, x) \mu_j(x) dx + \sum_k \int_0^\infty \bar{q}_k(s, x) \gamma_k(x) dx, \\
(12) \quad & \left[ s + \frac{\partial}{\partial x} + \lambda_j^* + \nu + \mu_j(x) \right] \bar{p}_j(s, x) = 0, \\
(13) \quad & \left[ s + \frac{\partial}{\partial x} + \mu_j(x) \right] \bar{p}_{ij}(s, x) = \lambda_i \bar{p}_j(s, x), \\
(14) \quad & \left[ s + \frac{\partial}{\partial x} + \gamma_k(x) \right] \bar{q}_k(s, x) = 0, \\
(15) \quad & \left[ s + \frac{\partial}{\partial x} + \gamma_k(x) \right] \bar{q}_{jk}(s, x) = 0, \\
(16) \quad & \bar{p}_j(s, 0) = \lambda_j \bar{p}_0(s) + \sum_{i \neq j} \int_0^\infty \bar{p}_{ji}(s, x) \mu_i(x) dx + \sum_k \int_0^\infty \bar{q}_{jk}(s, x) \gamma_k(x) dx,
\end{aligned}$$

$$(17) \quad \bar{p}_{ij}(s, 0) = 0,$$

$$(18) \quad \bar{q}_k(s, 0) = \nu_k \bar{p}_0(s),$$

$$(19) \quad \bar{q}_{jk}(s, 0) = \nu_k \bar{p}_j(s),$$

where  $\bar{\phantom{x}}$  represents the L. T. of the corresponding function.

Solving these equations (11) through (19), after some manipulation we obtain

$$(20) \quad \bar{p}_0(s) = \frac{1}{s - \bar{g}(s) + \lambda + \nu} \{1 + \sum_j \bar{p}_j(s, 0) \bar{f}_j(s + \lambda_j^* + \nu)\}$$

where  $\bar{g}(s) \equiv \sum_k \nu_k \bar{g}_k(s)$ ,

$$(21) \quad \bar{p}_j(s) = \int_0^\infty \bar{p}_j(s, x) dx = \bar{p}_j(s, 0) \cdot \frac{1 - \bar{f}_j(s + \lambda_j^* + \nu)}{s + \lambda_j^* + \nu},$$

$$(22) \quad \bar{p}_{ij}(s) = \int_0^\infty \bar{p}_{ij}(s, x) dx = \frac{\lambda_j}{\lambda_j^* + \nu} \bar{p}_j(s, 0) \left\{ \frac{1 - \bar{f}_j(s)}{s} - \frac{1 - \bar{f}_j(s + \lambda_j^* + \nu)}{s + \lambda_j^* + \nu} \right\} \quad (i \neq j),$$

$$(23) \quad \bar{q}_k(s) = \int_0^\infty \bar{q}_k(s, x) dx = \nu_k \bar{p}_0(s) \frac{1 - \bar{g}_k(s)}{s},$$

$$(24) \quad \bar{q}_{jk}(s) = \int_0^\infty \bar{q}_{jk}(s, x) dx = \nu_k \bar{p}_j(s) \frac{1 - \bar{g}_k(s)}{s},$$

Moreover, we get

$$(25) \quad \bar{p}_j(s, 0) = \frac{\lambda_j (\lambda_j^* + \nu) (s + \lambda_j^* + \nu) \alpha(s)}{(s + \lambda_j^* + \nu) \{(\lambda_j^* + \nu) + \lambda_j [\bar{f}_j(s) - \bar{f}_j(s + \lambda_j^* + \nu)]\} - (\lambda_j^* + \nu) \bar{g}(s) [1 - \bar{f}_j(s + \lambda_j^* + \nu)]}$$

where

$$(26) \quad \alpha(s) \equiv \bar{p}_0(s) + \sum_j \frac{1}{\lambda_j^* + \nu} \bar{p}_j(s, 0) [\bar{f}_j(s) - \bar{f}_j(s + \lambda_j^* + \nu)].$$

Using the following relation (by the total probability theorem);

$$(27) \quad \bar{p}_0(s) + \sum_j \bar{p}_j(s) + \sum_{\substack{i,j \\ (i \neq j)}} \bar{p}_{ij}(s) + \sum_k \bar{q}_k(s) + \sum_j \sum_k \bar{q}_{jk}(s) = \frac{1}{s},$$

we can calculate  $\alpha(s)$ . Then we obtain the L. T. of all state probabilities from (20)~(25). Since these may be (possibly) expressed in some complicated form, we consider the stationary availability hereafter. Now, from (25) we have

$$(28) \quad \lim_{s \rightarrow 0} s \bar{p}_j(s, 0) = \frac{\lambda_j(\lambda_j^* + \nu)}{\lambda - (\lambda_j - \nu) \bar{f}_j(\lambda_j^* + \nu)} \lim_{s \rightarrow 0} s \alpha(s).$$

Also, from (20) and (21), we have

$$(29) \quad \lim_{s \rightarrow 0} s \bar{p}_0(s) = \frac{1}{\lambda} \sum_j \bar{f}_j(\lambda_j^* + \nu) \lim_{s \rightarrow 0} s \bar{p}_j(s, 0),$$

$$(30) \quad \lim_{s \rightarrow 0} s \sum_j \bar{p}_j(s) = \sum_j \frac{1 - \bar{f}_j(\lambda_j^* + \nu)}{\lambda_j^* + \nu} \lim_{s \rightarrow 0} s \bar{p}_j(s, 0).$$

We can get the long-run availability of the system  $p_A(\infty)$  by applying the Tauberian theorem to equations (28)~(30), that is,

$$(31) \quad p_A(\infty) = \lim_{s \rightarrow 0} s \sum_{j=0}^m \bar{p}_j(s) = \lim_{s \rightarrow 0} s \alpha(s).$$

Obviously,

$$(32) \quad \lim_{s \rightarrow 0} s \left\{ \sum_k \bar{q}_k(s) + \sum_j \sum_k \bar{q}_{jk}(s) \right\} = \rho_B \lim_{s \rightarrow 0} s \alpha(s)$$

where

$$\rho_B \equiv \sum_k \nu_k \beta_k \quad \text{and} \quad \beta_k \equiv \int_0^\infty x g_k(x) dx \quad (\text{mean repair time for } B_k),$$

$$(33) \quad \lim_{s \rightarrow 0} s \sum_{\substack{i, j \\ (i \neq j)}} \bar{p}_{ij}(s) = \sum_j \frac{\lambda_j \lambda_j^*}{\lambda_j^* + \nu} \frac{(\lambda_j^* + \nu) \alpha_j - [1 - \bar{f}_j(\lambda_j^* + \nu)]}{\lambda - (\lambda_j - \nu) \bar{f}_j(\lambda_j^* + \nu)} \lim_{s \rightarrow 0} s \alpha(s)$$

where  $\alpha_j \equiv \int_0^\infty x f_j(x) dx$  (mean repair time for  $A_j$ ). By the above relations, we obtain the exact expression of the stationary availability  $p_A(\infty)$  as follows;

$$(34) \quad p_A(\infty) = \left[ 1 + \rho_B + \sum_j \frac{\lambda_j \lambda_j^*}{\lambda_j^* + \nu} \frac{(\lambda_j^* + \nu) \alpha_j - [1 - \bar{f}_j(\lambda_j^* + \nu)]}{\lambda - (\lambda_j - \nu) \bar{f}_j(\lambda_j^* + \nu)} \right]^{-1}.$$

By the way, if  $\lambda$  and  $\nu$  are sufficiently small, we have the approximation for (34). That is, if  $s$  is sufficiently small, then

$$\bar{f}_j(s) = \int_0^\infty \exp(-sx) f_j(x) dx = 1 - s \alpha_j + \frac{1}{2} s^2 \alpha_j^{(2)} - \dots$$

where  $\alpha_j^{(r)} \equiv (-1)^r \bar{f}_j(s)^{(r)}|_{s=0}$  and  $\alpha_j \equiv \alpha_j^{(1)}$ . Therefore, if  $\lambda_j^*, \nu \ll 1$ , then

$$(35) \quad \bar{f}_j(\lambda_j^* + \nu) \approx 1 - (\lambda_j^* + \nu) \alpha_j + \frac{1}{2} (\lambda_j^* + \nu)^2 \alpha_j^{(2)}.$$

Applying the above relation, we have the approximation for (34) as follows;

$$(36) \quad p_A(\infty) \approx \left[ 1 + \rho_B + \sum_j \lambda_j \lambda_j^* \frac{\alpha_j^{(2)}/2}{1 + (\lambda_j - \nu) [\alpha_j - (\lambda_j^* + \nu) \alpha_j^{(2)}/2]} \right]^{-1}.$$

This approximation is practical and useful for several type of the failure distribution which has both the 1st and 2nd moment.

(ii) Preemptive resume case

In this case, setting  $\delta=0$  in the equations (1)~(9), we can derive the stationary availability in the same way as the case of (i); we have

$$(37) \quad p_A(\infty) = \left[ 1 + \rho_B + \sum_j \lambda_j \frac{\lambda_j^* \alpha_j - [1 - \bar{f}_j(\lambda_j^*)]}{\lambda - \lambda_j \bar{f}_j(\lambda_j^*)} \right]^{-1}.$$

Note that the above (37) is given in conclusion by taking  $\nu=0$  in (34).

If  $\lambda_j^* \ll 1$ , then we have the approximation for (37) as follows;

$$(38) \quad p_A(\infty) \approx \left[ 1 + \rho_B + \sum_j \frac{\rho_j^{(2)}}{1 + \rho_j - \rho_j^{(2)}} \right]^{-1}$$

where  $\rho_j \equiv \lambda_j \alpha_j$  and  $\rho_j^{(2)} \equiv \frac{1}{2} \lambda_j \lambda_j^* \alpha_j^{(2)}$ .

#### 4. Time to system failure

In this section, we shall treat the time to system failure. We derive the exact expression of the mean time to system failure (MTSF) and its approximation formula. We introduce the following notations;

$\xi$ : time to system failure

$r_0(t)$ : probability that  $\xi > t$  and all units are operative at time  $t$ .

$r_j(t, x)dx$ : probability that  $\xi > t$ , and unit  $A_j$  is under repair and its e. r. t. lies between  $x$  and  $x+dx$ .

$r(t)$ : probability that at time  $t$  the system is in failure.

We can easily set up the following set of differential equations in order to obtain the MTSF;

$$(39) \quad \left[ \frac{d}{dt} + \lambda + \nu \right] r_0(t) = \sum_j \int_0^t r_j(t, x) \mu_j(x) dx,$$

$$(40) \quad \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda^* + \nu + \mu_j(x) \right] r_j(t, x) = 0,$$

$$(41) \quad \frac{d}{dt} r(t) = \nu r_0(t) + \sum_j (\lambda_j^* + \nu) r_j(t)$$

where  $r_j(t) \equiv \int_0^\infty r_j(t, x) dx$ .

The above equations are solved subject to the following boundary condition and initial condition.

$$(42) \quad r_j(t, 0) = \lambda_j r_0(t)$$

$$(43) \quad r_0(0) = 1.$$

Taking the Laplace transforms of (39)~(42), we can reduce them to

$$(44) \quad [s + \lambda + \nu] \bar{r}_0(s) = 1 + \sum_j \int_0^\infty \bar{r}_j(s, x) \mu_j(x) dx,$$

$$(45) \quad \left[ s + \frac{\partial}{\partial x} + \lambda_j^* + \nu + \mu_j(x) \right] \bar{r}_j(s, x) = 0,$$

$$(46) \quad s \bar{r}(s) = \nu \bar{r}_0(s) + \sum_j (\lambda_j^* + \nu) \bar{r}_j(s),$$

$$(47) \quad \bar{r}_j(s, 0) = \lambda_j \bar{r}_0(s).$$

Solving these equations, we have

$$(48) \quad \bar{r}_j(s) = \lambda_j \frac{1 - \bar{f}_j(s + \lambda_j^* + \nu)}{s + \lambda_j^* + \nu} \bar{r}_0(s),$$

$$(49) \quad \bar{r}_0(s) = [s + \nu + \sum_j (1 - \bar{f}_j(s + \lambda_j^* + \nu))]^{-1}.$$

The L. T. of the distribution function of time to first failure of the system is given by

$$(50) \quad \begin{aligned} \bar{\phi}(s) &= E[\exp(-s\xi)] = s \bar{r}(s) = \nu \bar{r}_0(s) + \sum_j (\lambda_j^* + \nu) \bar{r}_j(s) \\ &= \frac{\nu + \sum_j \lambda_j (\lambda_j^* + \nu) (1 - \bar{f}_j(s + \lambda_j^* + \nu)) / (s + \lambda_j^* + \nu)}{s + \nu + \sum_j \lambda_j (1 - \bar{f}_j(s + \lambda_j^* + \nu))}. \end{aligned}$$

Differentiating  $\bar{\phi}(s)$  with respect to  $s$ , and putting  $s=0$ , we obtain the exact expression for  $E(\xi)$ , the mean time to system failure, as

$$(51) \quad E(\xi) = - \frac{d}{ds} \bar{\phi}(s) \Big|_{s=0} = \frac{1 + \sum_j \lambda_j (1 - \bar{f}_j(\lambda_j^* + \nu)) / (\lambda_j^* + \nu)}{\nu + \sum_j \lambda_j (1 - \bar{f}_j(\lambda_j^* + \nu))}.$$

In the same way as in the previous section, if  $\lambda_j^*$  and  $\nu$  are sufficiently small, then we have the approximations of MTSF as follows;

1st kind approximation :

$$(52) \quad E(\xi) \approx \frac{1 + \sum_j \rho_j}{\nu + \sum_j (\lambda_j^* + \nu) \rho_j} = \left[ \nu + \frac{\sum_j \lambda_j^* \rho_j}{1 + \sum_j \rho_j} \right]^{-1}.$$

2nd kind approximation :

$$(53) \quad \begin{aligned} E(\xi) &\approx \frac{1 + \sum_j \{\rho_j - \lambda_j (\lambda_j^* + \nu) \alpha_j^{(2)} / 2\}}{\nu + \sum_j (\lambda_j^* + \nu) \{\rho_j - \lambda_j (\lambda_j^* + \nu) \alpha_j^{(2)} / 2\}} \\ &= \left[ \nu + \frac{\sum_j \lambda_j^* \{\rho_j - \lambda_j (\lambda_j^* + \nu) \alpha_j^{(2)} / 2\}}{1 + \sum_j \{\rho_j - \lambda_j (\lambda_j^* + \nu) \alpha_j^{(2)} / 2\}} \right]^{-1}. \end{aligned}$$

Setting  $\nu=0$  in (50) and (51),  $\bar{\phi}(s)$  and  $E(\xi)$  become the results which correspond to (14) and (15), respectively, Mine et al. [1].

### 5. Numerical example

In this section we apply the results of the preceding two sections to some particular cases. To give an impression of the accuracy of our approximation, we make some numerical comparisons for the system availability and MTSF. Four repair time distributions are considered; exponential, 2-Erlang, 10-Erlang, and regular distribution.

Now, we examine  $p_A(\infty)$  and  $E(\xi)$  for the 2-out-of-3 system, i.e., the case of  $m=3$ , with the following parameters;  $\lambda_1=0.01$ ,  $\lambda_2=0.02$ ,  $\lambda_3=0.025$ ,  $\mu_1=1.00$ ,  $\mu_2=2.00$ ,  $\mu_3=1.00$ ,  $\nu=0.01$  and  $\rho_B=0.05$ . These numerical results are given in the Table.

Table. Availability and MTSF

|                |          | Exponential | 2-Erlang | 10-Erlang | Regular  |
|----------------|----------|-------------|----------|-----------|----------|
| (Availability) |          |             |          |           |          |
| (i)            | Exact    | 0.951196    | 0.951480 | 0.951712  | 0.951771 |
|                | App.     | 0.951146    | 0.951454 | 0.951701  | 0.951762 |
| (ii)           | Exact    | 0.951196    | 0.951482 | 0.951716  | 0.951775 |
|                | App.     | 0.951157    | 0.951463 | 0.951707  | 0.951768 |
| (MTSF)         |          |             |          |           |          |
|                | Exact    | 87.5032     | 87.4021  | 87.3195   | 87.2987  |
|                | 1st App. | 87.0833     | 87.0833  | 87.0833   | 87.0833  |
|                | 2nd App. | 87.5216     | 87.4115  | 87.3236   | 87.3020  |

(i) : Preemptive repeat, (ii) : Preemptive resume

The numerical results show that (36) and (38) for the system availability and (52) or (53) for MTSF give good approximations.

### Acknowledgment

The author wishes to thank Dr. S. Takamatsu for his invaluable suggestions and encouragement.

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