

Temporal General Equilibrium in Overlapping Generations Model

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1. Introduction

We consider that in the competitive market, all the participants behave themselves as completely selfish and self-supported.

Samuelson [15] assumed that men enter the labor market at about the age of twenty and they work for forty-five years or so and then live for fifteen years in retirement. We are interesting in the old who are completely selfish and self-supported.

In a model, the old have no endowment, but his own fund with only money. There exists no bequest. There are two generations and the young have 2 life-spans and the old one life-span. We consider the temporal general equilibrium in an exchange economy. This type of model as developed in 1970's by Grandmont, Laroque and Younne [7, 8, 9, 10], although Grandmont has not shown the full model under uncertainty. In this paper, we exercise this task and prepair to extend it to a multi-period model. But we don't show the stationary equilibrium.

Consumption loan is possible if a trader borrows money only for one period. However, a retired consumer cannot borrow any money because of the assumption of no endowment. Since we are interesting in the existence of equilibrium in the market with private pension, we shall focus on the function of the fund without banking. On behalf of no borrowing and lending, there is no interest for money hoarding.

2. Definitions and Assumptions of a Model

We consider an exchange economy, where trade starts at period $t=1$ and continues sequentially over time. A trader who initially enters into a market works for one period and lives for one period after his retirement. Each trader saves his or

her own fund for the retirement life period with no bequest. Each generation is denoted by an index $a=1, 2$. If $a=1$, the generation is young. The population of each generation is the same and constant.

At each period there exist l perishable goods and paper money. A young trader who works for one remaining period is endowed with $e_{1l} \in R_+^l$ which is perfectly foresighted, where R_+^l is a closed positive cone of R^l . At period $t=1$, every trader has no debt. The old are not endowed, but withdraw their fund m_{20} , which has been saved for the working period. For simplicity, all the traders cannot borrow and lend money from the others. We assume

Assumption 2.1 Let $e_{1l} \gg 0$ and $m_{20} > 0$.

For a young trader, let $x_1 = (x_{11}, x_{12}) \in R_+^{2l}$ be a consumption stream. Let $m_{11} \in R_+$ be money holding. For an old trader, let $x_{21} \in R_+^l$ be a consumption stream. At period $t=1$, money price is one and a vector of market prices is denoted as $p_1 \in P_1 = \text{Int } R_+^l$, where p_1 is a vector of monetary market prices of perishable goods. At period $t=2$, a vector of market prices is denoted as $p_2 \in P_2 = \text{Int } R_+^l$.

An action is a decision to trade in a commodity market and a money market at period 1. We denote the action by $a_1 = (x_{11}, m_{11}) \in R_+^{l+1}$. A plan is a decision to trade in a commodity market and a money market at period 2. We assume that consequences of his consumption behavior at period $t=1$ are under certainty and ones at period 2 are under uncertainty. We adopt the expected utility hypothesis with respect to the consumer's preferences to the consumption stream.

A space of consequences of the trader's action and plan is the space of consumption stream $C = R_+^{2l}$. Let $M(R_+^l)$ be the space of all probability measures on the measures on the measurable space $(R_+^l, B(R_+^l))$, where $B(R_+^l)$ is the Borel σ -field on R_+^l . On the Cartesian product $M^2(R_+^l)$ of the spaces $M(R_+^l)$, the consumer chooses a probability measure and the preferences are represented by a preordering $\succeq^l$. Thus we assume as follows [6, Assumption 1, 6] :

Assumption 2.2 There exists a von Neumann-Morgenstern utility $u : C \rightarrow R$ which is continuous, bounded, such that the mapping $v : M^2(R_+^l) \rightarrow R$ defined by $v(\mu) = \int_C u \, d\mu$ for any $\mu \in M^2(R_+^l)$ is a representation of the preferences $\succeq^l$. Any von Neumann-Morgenstern utility u is concave and monotone.

Under Assumption 2.2, the young trader is faced on the certainty at period 1 and

the uncertainty at $t=2$. Then, according to [13], a mapping $v(\mu)$ has the following form for any $\mu = (\mu_1, \mu_2) \in M^2(R_+^l)$.

$$\textbf{Assumption 2.3} \quad v(\mu) = u_1 + \int_{R_+^l} u_2 \, d\mu_2.$$

The uncertainty in Assumptions 2.2 and 2.3 means one of consequences when the consumer will act in the future period. On the other hand, the consumer expects the next period's equilibrium price system. According to Grandmont's exogenous expectation [6, Assumption 2], we assume a Grandmont expectation function that the present price system quoted $p_1 \in P_1$ is correspondent to a probability distribution $\phi(p_1) \in M(P_2)$.

Assumption 2.4 The expectation function $\phi(p_1) : P_1 \rightarrow M(P_2)$ is continuous, when $M(P_2)$ is endowed with the topology of weak convergence of probability measures.

Since we introduce money to the model, according to Grandmont [5, (c. 3), p. 222] we assume a further assumption.

Assumption 2.5 The set $\{\phi(p_1) \mid p_1 \in P_1\}$ is relatively (weakly) compact.

Well, we shall show that two uncertainties of consequences of consumer's actions at the future period and expectation of spot price system at the future period are related by the assumption of a random variable γ .

A young trader's action is defined as a decision of trade in markets at period 1 and represented by $a_{11} = (x_{11}, m_{11}) \in R_+^{l+1}$. A plan is defined as a decision of an action at $t=2$ and represented by $a_{12} = x_{12} \in R_+^l$. First, given a price vector p_1 and an endowment e_{11} , the consumer chooses an action restricted by a subset of an action space $A_1 = R_+^{l+1}$, which is denoted by $\beta_{11}(p_1, e_{11}) = \{a_{11} \in A_1 \mid p_1 \cdot a_{11} \leq p_1 \cdot e_{11}\}$. Second, a plan is restricted by each subset β_{12} of the plan space A_2 which depends on the chosen action a_{12} . This subset is represented as follows: $\beta_{12}(p_2 \mid p_1, a_{11}) = \{a_{12} \in A_2 = R_+^l \mid p_2 \cdot a_{12} \leq m_{11}\}$.

We assume that the young trader has the following continuous linear mapping γ , which we call a transformation function.

Assumption 2.6 $\gamma: a_{11} \bullet a_{12}(P_2 \times R_+) \rightarrow C$, where $a_{12}(P_2 \times R_+)$ denotes the range of a measurable function $a_{12}: P_2 \times R_+ \rightarrow A_2$.

Since the transformation function γ is a random variable because a price vector p_2 is a random variable by Assumption 2.3, a probability distribution defined on the space of consequences from the space of price expectation P_2 can be induced by the random variable γ . According to [13], a von Neumann-Morgenstern utility function is denoted as $u(\gamma(a_{11} \bullet a_{12})) = u_1(x_{11}) + u_2(a_{12}(p_2, m_{11}))$ by the function γ . Therefore, a conditional expected utility function on a_{11} and p_1 , $v(a_{11}, p_1)$ is denoted by

$$v(a_{11}, p_1) = u_1(x_{11}) + \int_{R_+^I} u_2(a_{12}(\bullet, m_{11})) d\phi(p_1). \quad 2.1$$

By the relation 2.1, a rule of Grandmont's decision making [6, p. 884] is formulated as follows:

Given p_1 , for any actions $a_{11}, a'_{11} \in \beta_{11}(p_1, e_{11})$, $a_{11} \stackrel{S}{\succeq}_{p_1} a'_{11}$ if and only if $v(a_{11}, p_1) \geq v(a'_{11}, p_1)$. 2.2

On the other hand, an old trader's action is defined as a decision of trade in markets at period 1 and represented by $a_{21} = x_{21} \in R_+^I$. There is no plan for the old trader at $t=2$ because he will die without bequest. Given a price vector p_1 and money m_{20} , the consumer chooses an action restricted by a subset of an action space $A_{21} = R_+^I$, which is denoted by $\beta_{21}(p_1, m_{20}) = \{a_{21} \in A_{21} \mid p_1 \bullet a_{21} \leq m_{20}\}$.

3. Generations' Optimality

Each trader decides on an action and a plan. Since there exist two generations in our model, we shall show each generation's optimality and each demand correspondence's properties.

3.1 The young generation's optimality

The young trader decides on an action a_{11} and a plan a_{12} at period 1 subject to the sets of his budgets $\beta_{11}(p_1, w_1)$, $\beta_{12}(p_2, m_{11})$ with two steps. In the first step, we shall solve a problem to decide on the plan a_{12} , given the price p_1 and the action a_{11} , subject to the set of budget $\beta_{12}(p_2, m_{11})$. When we substitute the solution a_{12}^* for the equation 2.2, we shall obtain a conditional expected utility function $v(a_{11}, p_1)$. In the second step, we shall solve the action a_1 subject to the set of budget $\beta_{11}(p_1, w_1)$.

First, we show properties of β_{11}, β_{12} according to the results by Debreu [3]. Next,

we solve the problem of plan-decision and prove the properties of the conditional expected utility function $v(a_{11}, p_1)$. Then, we obtain the solution of action-problem. Finally, we define the i th young trader' excess demand correspondence ξ_i^1 and prove the properties of ξ_i^1 .

For a young trader, we assume that consumption sets $A_1, A_2 \subset R_+^L$ are non-empty and compact. We define a correspondence $\beta_{11}(p_1, w_1) = \{a_{11} \in A_1 \mid p_1 \cdot x_1 + m_{11} \leq p_1 \cdot e_{11}\}$ from $P_1 \times R_+$ to A_1 and a correspondence $\beta_{12}(p_2, w_2) = \{a_{12} \in A_2 \mid p_2 \cdot a_{12} \leq m_{11}\}$ from $P_2 \times R_+$ to A_2 . By Debreu [3], the correspondences have the following properties :

Lemma 3.1 Correspondences β_{11}, β_{12} are non-empty, convex-valued, compact-valued and continuous on $P_1 \times R_+$ and $P_2 \times R_+$, respectively.

Proof For any $(p_1, w_1) \in P_1 \times R_+$, the correspondence β_{11} is non-empty, convex-valued, compact-valued from the form of the β_{11} and the assumptions of the consumption set A_1 . According to Debreu [3, Lemma 3], if A_1 is non-empty, compact, and convex, and $w_1^0 > \min p_1^0 \cdot A_1$, then the correspondence β_{11} is continuous at (p_1^0, w_1^0) . For any $(p_2^0, m_{11}^0) \in P_2 \times R_+$, the correspondence β_{12} is also non-empty, convex-valued, compact-valued. Furthermore, β_{12} is also continuous at (p_2^0, m_{11}^0) . $\square$

Next, given the price p_1 and the action a_{11} at period 1, we solve a well known problem to decide on the plan a_{12} subject to the set of budget $\beta_{12}(p_2, m_{11})$.

Problem 3.1 Under Assumption 2.1, $p^2 \gg 0$, given the current consumption x_{11} and money m_{11} ,

$$\begin{aligned} \max_{\{x_{12}\}} \quad & u_{11}(x_{11}) + u_{12}(x_{12}) \\ \text{subject to } & \beta_{12}(p_2, m_{11}) = \{x_{12} \in A_2 \mid p_2 \cdot x_{12} \leq m_{11}\}. \end{aligned}$$

Solution The von Neumann-Morgenstern utility functions u_{11} and u_{12} are real-valued and continuous by Assumption 2.2. Since u_{12} is continuous and β_{12} is compact, the function u_{12} has its maximum on β_{12} . $\square$

Let x_{12}^* be a solution for Problem 3.1. In Section 2, we have defined a conditional expected utility function $v(a_{11}, p_1)$. When we substitute the solution x_{12}^* for the equation 2.2, we shall obtain a conditional expected utility function $v(a_{11}, p_1) = u_{11}(x_{11}) + \int u_{12}^*(\cdot, m_{11}) d\phi(p_1)$. Here, we prove the properties of the function v .

Theorem 3.2 The conditional expected utility function $v(a_{11}, p_1)$ is (i) bounded and continuous, for a_{11}, p_1 , (ii) concave and strictly monotone for a_{11} .

Proof (i) Since Assumption 2.2, the von Neumann-Morgenstern utility function u is bounded, v is bounded. We prove that v is continuous. Let a sequence $\{(a_{11}^k, p_1^k)\}$ converge to (a_{11}^0, p_1^0) in $A_1 \times P_1$. Now, the utility functions $u_{11}(x_{11}^k)$ and $u_{12}^*(\cdot, m_{11}^k)$ are uniformly bounded and converge continuously to $u_{11}(x_{11}^0)$ and $u_{12}^*(\cdot, m_{11}^0)$. Since by Assumption 2.4, an expectation function $\phi(p_1)$ is continuous, a sequence $\{\phi(p_1^k)\}$ converges weakly to $\phi(p_1^0)$. Therefore, $\lim v(a_{11}^k, p_1^k) = v(a_{11}^0, p_1^0)$. (ii) The concavity of the function v is obtained by Sondermann [16, Lemma 7.2], since by Assumption 2.2, u_{11} and $\int u_{12} d\phi$ are concave and the sum of concave functions is concave. The strictly monotone of v is also evident by Assumption 2.2. $\square$

Next, we shall solve the action a_1 , subject to the set of budget $\beta_{11}(p_1, w_1)$.

Problem 3.2 Under Assumption 2.1, $p_1 \gg 0$,

$$\begin{aligned} \max_{(x_{11}, m_{11})} \quad & u_{11}(x_{11}) + \int u_{12}^*(\cdot) d\phi(p_1) \\ \text{subject to} \quad & \beta_{11}(p_1, w_1) = \{a_{11} \in A_1 \mid p_1 \cdot x_{11} + m_{11} \leq p_1 \cdot e_{11}\}. \end{aligned}$$

Solution The conditional expected utility function v is real-valued and continuous by Assumption 2.2. Since v is continuous and β_{11} is compact, the function v has its maximum on $\beta_{11}(p_1, w_1)$. $\square$

A rule of Grandmont's decision making is that given p_1 , for any actions a_{11} and $a'_{11} \in \beta_{11}(p_1, w_1)$, $a_{11} \succ_{p_1} a'_{11}$ if and only if $v(a_{11}, p_1) \geq v(a'_{11}, p_1)$. Therefore, maximizing v on $\beta_{11}(p_1, w_1)$ yields the set $\xi_1^i(p_1) = \{a_{11} \in A_1 \mid v(a_{11}, p_1) \geq v(a'_{11}, p_1) \text{ for every } a'_{11}\}$. We call $\xi_1^i(p_1)$ his demand correspondence. The demand correspondence $\xi_1^i(p_1)$ has the following properties.

Lemma 3.3 ξ_1^i is nonempty, convex-valued, compact-valued and upper hemicontinuous. For all $p_1 \in P_1$ and all $a_{11} \in \xi_1^i$, $p_1 \cdot x_{11} + m_{11} = p_1 \cdot e_{11}$.

Proof By Lemma 3.1, ξ_1^i is nonempty, convex-valued, compact-valued and upper hemicontinuous. From the monotonicity of preferences, his budget satisfies

the equality [16, Proposition 7.4 and Corollary]. $\square$

By Grandmont [5, Proposition 4.2], the young trader' demand correspondence ξ^i satisfies a boundary condition.

Theorem 3.4 For any $a_{11}^i \in \xi^i(p^i)$, if p^i tends to $p^0 \in R_+^L / P_1$, or if $\|p^i\|$ tends to $+\infty$, then $\|a_{11}^i\|$ tends to $+\infty$.

Proof Assume the contrary. There exists a subsequence that a_{11}^i converges to $a_{11}^0 = (x_{11}^0, m_{11}^0)$. According to Assumption 2.4, there exists an element $\phi(p^0)$ that $\phi(p^i)$ converges to $\phi(p^0)$. We define a conditional expected utility function h by $h(a_{11}) = u_{11}(x_{11}) + \int u_{12}^*(\cdot) d\phi(p^0)$. From Theorem 3.2, for any a_{11} , $\lim v(a_{11}, p^i) = h(a_{11})$, $\lim v(a_{11}^i, p^i) = h(a_{11}^0)$, and h is continuous.

(i) p^i tends to $p^0 \in R_+^L / P_1$.

$p_1^0 \cdot x_{11}^0 + m_{11}^0 \leq p_1^0 \cdot e_{11}$ and $p_1^0 \cdot e_{11} > 0$. Since a_{11}^0 is optimal, $h(a_{11}^0) \geq h(a_{11})$ for all a_{11} such that $p_1^0 \cdot x_{11} + m_{11}^0 \leq p_1^0 \cdot e_{11}$. But there exists an element $p_{1k}^0 = 0$. Therefore it contradicts since its k th good increased a_{11} is also optimal.

(ii) $\lim \|p^i\| = +\infty$.

Let $\pi^j = p^j / \|p^j\|$ and π^j converge to some $\pi^0 > 0$. Since m_{11}^i is bounded and $(p^i / \|p^i\|) \cdot x_{11}^i + m_{11}^i / \|p^i\| \leq (p^i / \|p^i\|) \cdot e_{11}$, $\pi^0 \cdot x_{11}^0 \leq \pi^0 \cdot e_{11}$. Since $\pi^0 \cdot e_{11} > 0$, there exists an element $\underline{x}_{11}$ such that $\pi^0 \cdot \underline{x}_{11} < \pi^0 \cdot e_{11}$. Define $x_{11}^\lambda = \lambda \underline{x}_{11} + (1-\lambda)x_{11}^0$ for any $\lambda \in [0, 1]$. Then, $a_{11}^\lambda = (x_{11}^\lambda, m_{11}) \in A_1$ and $\pi^0 \cdot x_{11}^\lambda < \pi^0 \cdot e_{11}$. When we take j large enough, $p^j \cdot x_{11}^\lambda + m_{11} < p^j \cdot e_{11}$. Therefore, $v(a_{11}^\lambda, p^j) \geq v(a_{11}^j, p^j)$. In the limit, $h(a_{11}^0) \geq h(a_{11}^\lambda)$. If λ converges to 0, $h(a_{11}^0) \geq h(a_{11})$. If $a_{11} = (x_{11}^0, m_{11})$ such that $m_{11} > m_{11}^0$, the price of money is zero in the limit. Then, $h(a_{11}) > h(a_{11}^0)$. This contradicts the optimality of a_{11}^0 . $\square$

3.2 The old generation's optimality

For an old trader, we assume that a consumption set $A_1 \subset R_+^L$ is non-empty and compact. We define a correspondence $\beta_{21}(p_1, m_{20}) = \{a_{21} \in A_1 \mid p_1 \cdot a_{21} \leq m_{20}\}$ from $P_1 \times R_+$ to A_1 . The old trader decides on an action a_{21} at period 1, subject to the set of his budget $\beta_{21}(p_1, m_{20})$.

Theorem 3.5 Correspondence β_{21} is convex-valued, compact-valued and upper hemi-continuous on $P_1 \times R_+$.

Proof See Lemma 3.1. □

Since the old trader does not face on the uncertainty at period 2, his problem to obtain an optimal consumption at period 1 is well known. We assume that u_{21} is continuous, concave, bounded real-valued utility function.

Problem 3.3 Under Assumption 2.1, $p_1 \gg 0$, given money $m_{20} > 0$,

$$\begin{aligned} \max_{\{x_{21}\}} \quad & u_{21}(x_{21}) \\ \text{subject to} \quad & \beta_{21}(p_1, m_{20}) = \{x_{21} \in A_1 \mid p_1 \cdot x_{21} \leq m_{20}\}. \end{aligned}$$

Solution The ordinal utility function u_{21} is real-valued and continuous. Since u is continuous and β_{21} is compact, the function u_{21} has its maximum on β_{21} . □

For the i th old trader, maximizing u_{21}^i on $\beta_{21}(p_1, m_{20})$ yields the set $\xi_2^i(p_1) = \{a_{21} \in A_1 \mid \text{for every } a'_{21}, \text{ one has } u_{21}(a_{21}) \geq u_{21}(a'_{21})\}$. The demand correspondence $\xi_2^i(p_1)$ has the following properties.

Lemma 3.6 ξ_2^i is nonempty, convex-valued, compact-valued and upper hemi-continuous. For all $p_1 \in P_1$ and all $a_{21} = x_{21} \in \xi_2^i$, $p_1 \cdot x_{21} = p_1 \cdot m_{20}$.

Proof By Lemma 3.1, ξ_2^i is nonempty, convex-valued, compact-valued and upper hemicontinuous. □

Theorem 3.7 For any $a_{21}^i \in \xi_2^i(p_1^i)$, if p_1^i tends to $p_1^0 \in R_+^1 / P_1$, then $\|a_{21}^i\|$ tends to $+\infty$.

Proof See Debreu [3, Lemma 4]. □

4. Market Equilibrium

We shall show the existence of an equilibrium in overlapping generations model. In an exchange economy, there are n young traders and n old traders. An allocation

is a $2n$ -tuple $a = (a_{11}^i, \dots, a_{1n}^i, a_{21}^i, \dots, a_{2n}^i) \in R_+^{n(2l+1)}$, where $a_{11}^i = (x_{11}^i, m_{11}^i)$ and $a_{21}^i = x_{21}^i$ ($i=1, \dots, n$) for all i . $\sum_{i=1}^n (x_{11}^i + x_{21}^i - e_{11}^i) = 0$ and $\sum_{i=1}^n m_{20}^i = M > 0$. An equilibrium price system is a price system p_1^* of P_1 such that there exists an allocation $a^* = (a_{11}^{1*}, \dots, a_{1n}^{n*}, a_{21}^{1*}, \dots, a_{2n}^{n*})$ with $a_{11}^{i*} \in \xi_1^i(p_1^*)$ and $a_{21}^{i*} \in \xi_2^i(p_1^*)$ for all i .

In Section 3, we obtained the i th young trader's demand correspondence ξ_1^i and the i th old trader's demand correspondence ξ_2^i . For all $p_1 \in P_1$, we define an excess demand correspondence ζ as follows:

$$\zeta(p_1) = \sum_{i=1}^n \{(\xi_1^i(p_1) - e_{11}^i) + (\xi_2^i(p_1) - m_{20}^i)\}.$$

Theorem 4.1 (Grandmont's Market Equilibrium Lemma)

Let $\bar{S} = \{p_1 \in R_+^l \mid \sum_{i=1}^l p_{1i} = 1\}$. Let S be any subset of $\bar{S}$ containing $S^0 = \{p_1 \in S \mid p_1 \gg 0\}$. If ζ is a nonempty-valued correspondence whose graph is closed in $S \times R_+^l$ and which satisfies:

- (1) ζ is Γ_1 , compact-valued and upper hemi-continuous on S ,
- (2) strong Walras' Law: $p_1 \cdot \zeta(p_1) = 0$, for all $p_1 \in S$,
- (3) a boundary condition: if p_1^j tends to $p_1^0 \in \bar{S}/S$ and for any sequence $a^j \in \zeta(p_1^j)$, there exists $\bar{p}_1$ in S such that $\bar{p}_1 \cdot a^j > 0$ for infinitely many j , then there exists p_1^* in S such that $0 \in \zeta(p_1^*)$.

Proof See Debreu [3, Lemma 4] and Grandmont [5, 6]. □

Theorem 4.2 Under Assumptions 2.1, ..., 2.6, there exists an equilibrium price system p_1^* .

Proof See Grandmont [5, Theorem 5.1]. □

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